Subject for this video:

Using Properties of the Definite Integral

Reading:

- **General:** Section 5.4 The Definite Integral
- **More Specifically:** Pages 364 - 365, Examples 3,4

Homework: H77: Using Properties of the Definite Integral 5.4#33,41,45,49,51,53

Recall the Definition fo the Definite Integral from the previous previous video

Definition of the *Definite Integral and Signed Area*

Words: The definite integral of $f(x)$ from $x = a$ to $x = b$.

Symbol:

$$\int_{x=a}^{x=b} f(x)dx$$

Alternate Words: The *signed area* of the region between the graph of $f(x)$ and the x axis on the interval $[a, b]$.

Alternate Symbol: SA

Usage: $f(x)$ is continuous on the interval $[a, b]$.

Meaning: the number $\lim_{n\to\infty} L_n$ (which is also the value of $\lim_{n\to\infty} R_n$)

That is,

$$SA = \int_{x=a}^{x=b} f(x)dx \stackrel{\text{def}}{=} \lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n$$

.

[Example 1] The graph of $f(x)$ is shown.

(unsigned areas)

The areas of the six shaded regions are:

The area of region A is 6.

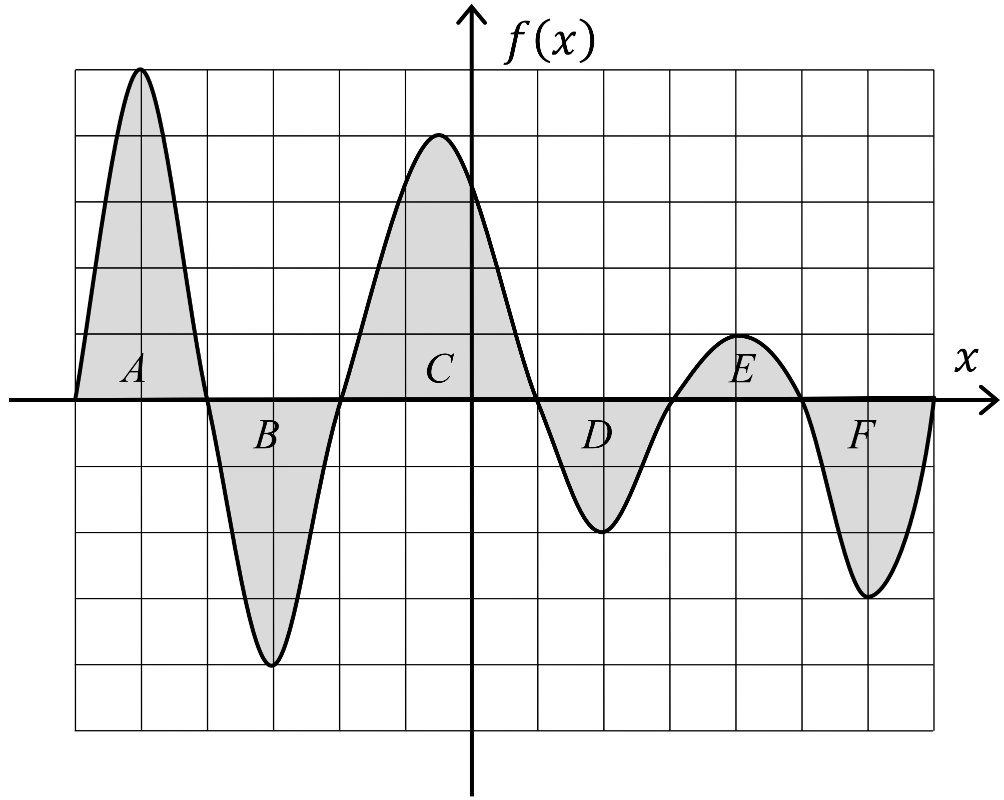
The area of region B is 5.

The area of region C is 7.

The area of region D is 3.

The area of region E is 2.

The area of region F is 4.



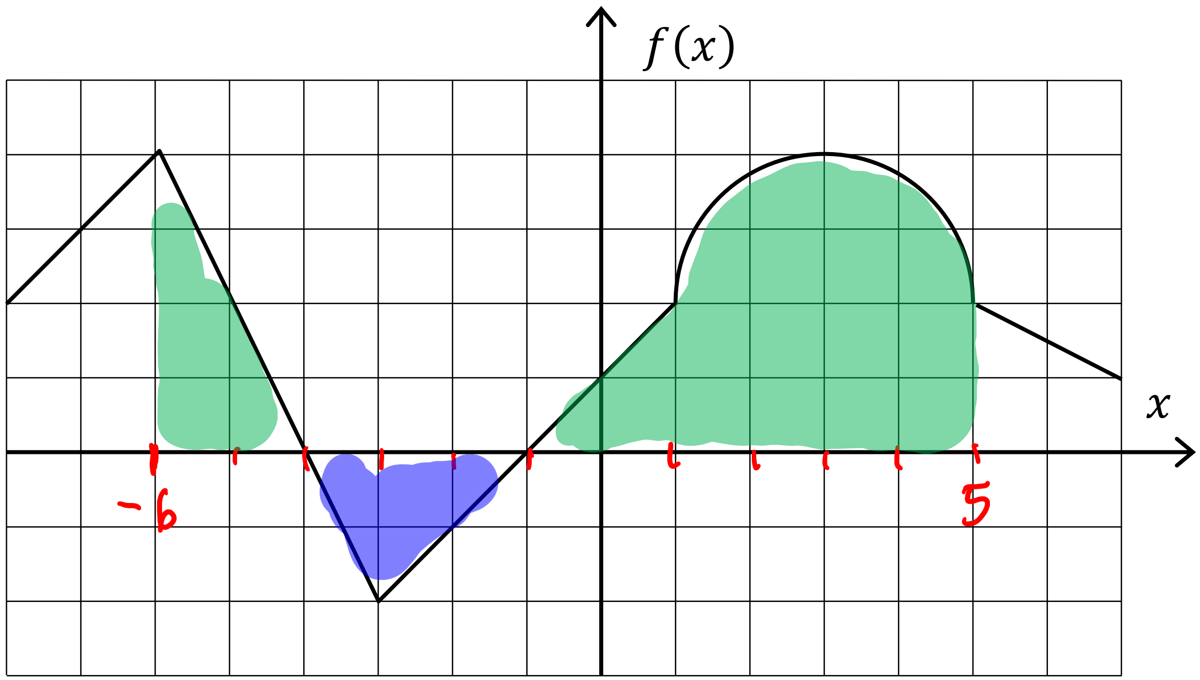
Find the value of the definite integrals.

$$\int_{x=-4}^{x=3} f(x)dx = -B + C - D = -5 + 7 - 3 = -1 = SA$$

signed area

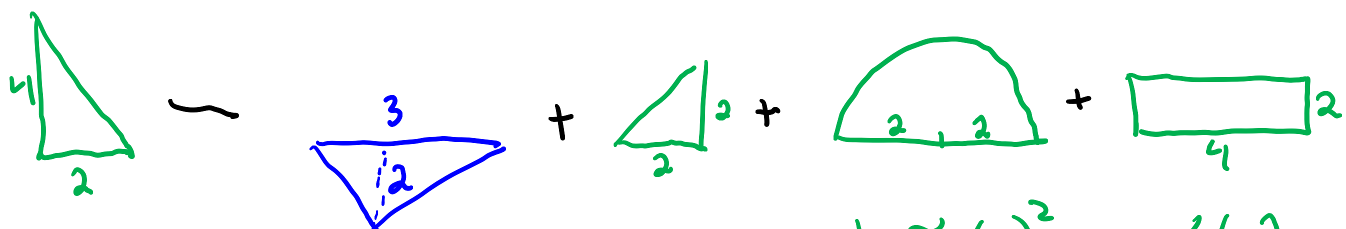
$$\int_{x=-2}^{x=5} f(x)dx = C - D + E = 7 - 3 + 2 = 6 = SA$$

[Example 2] The graph of $f(x)$ is shown.



Shade the region corresponding to the definite integral and find the value of the integral

$$\int_{x=-6}^{x=5} f(x)dx =$$



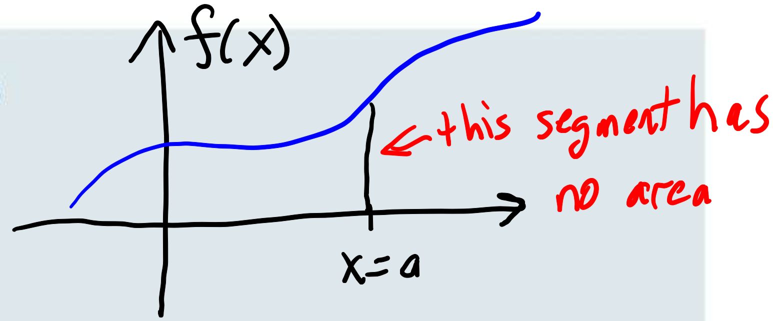
$$= \frac{1}{2}\cdot 2\cdot 4 - \frac{1}{2}\cdot 3\cdot 2 + \frac{1}{2}\cdot 2\cdot 2 + \frac{1}{2}\cdot \pi\cdot (2)^2 + 4\cdot 2$$

$$= 4 - 3 + 2 + 2\pi + 8$$

$= 11 + 2\pi$ exact answer

≈ 17.28 decimal approximation

PROPERTIES Properties of Definite Integrals



1. $$\int_a^a f(x)\,dx = 0$$

2. $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

3. $$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx, \ k \text{ a constant}$$

4. $$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

5. $$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

[Example 3] Given the values of the following definite integrals,

$$\int_{1}^{4} x dx = 0.75 \quad and \quad \int_{1}^{4} x^2 dx = 21 \quad and \quad \int_{4}^{5} x^2 dx = \frac{61}{3}$$

compute the following integrals.

(*a*) $\int_{x=1}^{x=4} (7x - 2x^2) dx$ $= 7\int_{x=1}^{x=4} x\,dx - 2\int_{x=1}^{x=4} x^2\,dx$

Properties 3+4

$= 7(0.75) - 2(21)$

$= -57.75$

$$(\boldsymbol{b}) \int_{x=1}^{x=5} -4x^2\,dx = -4\int_{x=1}^{x=5} x^2\,dx$$

property 3

$$= -4\left[\int_{x=1}^{x=4} x^2\,dx + \int_{x=4}^{x=5} x^2\,dx\right]$$

property 5

$$= -4\left[21 + \frac{61}{3}\right]$$

$$= -4\left[\frac{63}{3} + \frac{61}{3}\right]$$

$$= -4\left[\frac{124}{3}\right]$$

$$= -\frac{496}{3}$$ exact answer

$$\approx -165.33$$ decimal approximation

(c) $\displaystyle\int_{x=5}^{x=5} (285 - 17x + 23x^2)^{13}\,dx = 0$

property 1

(d) $\displaystyle\int_{x=4}^{x=1} (7x - 2x^2)\,dx = -\int_{x=1}^{x=4} (7x - 2x^2)\,dx = -(-57.75) = \boxed{57.75}$

property 3

result from (a)