Subject for this video:

Definite Integrals Involving Substitution

Reading:

- **General:** Section 5.5 The Fundamental Theorem of Calculus
- **More Specifically:** Pages 372 - 373, Examples 3, 4

Homework: H79: Definite Integrals Involving Substitution (5.5#37,39,45)

The *Fundamental Theorem of Calculus* (*FTC*)

(the relationship between ***definite integrals*** and ***antiderivatives***)

If $f(x)$ is continuous on the interval $[a, b]$, then

$$\int_a^b f(x)dx \underset{FTC}{=} \left(\int f(x)dx\right)\Bigg|_a^b$$

The Substitution Method for finding the *indefinite integral*

$$F(x) = \int f(x)\,dx$$

where the integrand $f(x)$ involves a *nested function*.

Step 1 Identify the inner function and call it $\boldsymbol{u}$. Write the equation $inner(x) = u$ to introduce the single letter u to represent the inner function. Circle the equation.

Step 2 Build the equation $\boldsymbol{dx = \frac{1}{u'}du}$. To do this, first find u', then use it to build equation $dx = \frac{1}{u'}du$. Circle the equation.

Step 3 Substitute, Cancel, Simplify. In steps (1) and (2) you have two circled equations. Substitute these into the integrand of your indefinite integral. Cancel as much as possible and simplify by using the *Constant Multiple Rule*. The result should be a new basic integral involving just the variable u. (See **Remarks about Step 3** on the next page.)

Step 4 Integrate. Find the new indefinite integral by using the indefinite integral rules. The result should be a function involving just the variable u (with constant of integration $+C$).

Step 5 Substitute Back. Substitute $u = inner(x)$ into your function from Step (4) The result will be a new function of just the variable x. (Be sure to include the constant of integration $+C$ in your result.) This is the $F(x)$ that we seek.

[Example 1](similar to 5.5#37) find $\int_0^1 3x(x^2-1)^4 dx$

(Give an exact answer in the form of an integer or a simplified fraction.)

$$\int_0^1 3x(x^2-1)^4 dx \underset{FTC}{=} \left(\int 3x(x^2-1)^4 dx \right)\Big|_0^1$$

$$= \left(\frac{3(x^2-1)^5}{10} + D \right)\Big|_0^1$$

$$= \left(\frac{3((1)^2-1)^5}{10} + \cancel{D} \right) - \left(\frac{3((0)^2-1)^5}{10} + \cancel{D} \right)$$

$$= \frac{3(0)^5}{10} - \frac{3(-1)^5}{10}$$

$$= 0 - \frac{3(-1)}{10}$$

$$= \frac{3}{10}$$

Indefinite Integral Details

Find $F(x) = \int 3x(x^2-1)^4 dx$

Step1 $x^2-1 = u$

Step2 $u' = 2x$, so $dx = \frac{1}{2x} du$

Step3
$$\int 3x(x^2-1)^4 dx = \int 3x(u)^4 \frac{1}{2x} du$$
$$= \int \frac{3}{2} u^4 du$$
$$= \frac{3}{2} \int u^4 du$$

Step4 $\frac{3}{2}\int u^4 du = \frac{3}{2}\left(\frac{u^5}{5} + c\right) = \frac{3u^5}{10} + D$

Step5 $F(x) = \int 3x(x^2-1)^4 dx = \frac{3(x^2-1)^5}{10} + D$

[Example 2](similar to 5.5#39) find $\int_4^9 \frac{2}{x-3}\,dx$

(Give an exact answer and a decimal approximation, rounded to three decimal places.)

$$\int_4^9 \frac{2}{x^2-3}\,dx \underset{FTC}{=} \left(\int \frac{2}{x-3}\,dx\right)\Big|_4^9$$

$$= \left(2\ln(|x-3|)+C\right)\Big|_4^9$$

$$= \left(2\ln(|(9)-3|)+\cancel{C}\right) - 2\ln(|(4)-3|)+\cancel{C}$$

$$= 2\ln(|6|) - 2\ln(|1|)$$

$$= 2\ln(6) - 2\cancel{\ln(1)} \to 0$$

$$= 2\ln(6) \qquad b\ln(a) = \ln(a^b)$$

$$= \ln(36) \quad \text{exact answer}$$

$$\approx 3.584 \quad \text{decimal approximation}$$

Indefinite Integral Details

Find $F(x) = \int \frac{2}{x-3}\,dx$

Step 1 $\quad x-3 = u$

Step 2 $\quad u' = \frac{d}{dx}\, x-3 = 1$, so $\quad dx = \frac{1}{1}\,du$

$$dx = du$$

Step 3 $\quad \int \frac{2}{x-3}\,dx = \int \frac{2}{u}\,du = 2\int \frac{1}{u}\,du$

Step 4 $\quad 2\int \frac{1}{u}\,du = 2\ln(|u|) + C$

Step 5

$$F(x) = 2\ln(|x-3|) + C$$

[Example 3](similar to 5.5#45) find $\int_0^1 xe^{(-x^2)}dx$

(Give an exact answer and a decimal approximation, rounded to three decimal places.)

$$\int_0^1 xe^{(-x^2)}dx \underset{FTC}{=} \left(\int xe^{-x^2}dx\right)\Big|_0^1$$

$$= \left(\frac{-1}{2}e^{(-x^2)}+C\right)\Big|_0^1$$

$$= \left(-\frac{1}{2}e^{(-(1)^2)}+\cancel{C}\right)$$

$$-\left(-\frac{1}{2}e^{(-(0)^2)}+\cancel{C}\right)$$

$$= \frac{-1}{2}e^{(-1)}+\frac{1}{2}e^{(0)}$$

$$= -\frac{1}{2}\cdot\frac{1}{e}+\frac{1}{2}\cdot 1$$

$$= \frac{1}{2}-\frac{1}{2e}$$

$$= \frac{e-1}{2e} \quad \text{exact answer}$$

$$\approx 0.316 \quad \text{decimal approximation}$$

Indefinite Integral Details

Find $F(x) = \int xe^{(-x^2)}dx$

Step 1 $\quad -x^2 = u$

Step 2 $\quad u' = \frac{d}{dx}-x^2 = -2x$

so $dx = \frac{-1}{2x}du$

Step 3 $\quad \int xe^{(-x^2)}dx = \int xe^{(u)}\frac{-1}{2x}du$

$$= \int \frac{-1}{2}e^{(u)}du = \frac{-1}{2}\int e^{(u)}du$$

Step 4 $\quad \frac{-1}{2}\int e^{(u)}du = \frac{-1}{2}e^{(u)}+C$

Step 5 $\quad F(x) = \int xe^{(-x^2)}dx = \frac{-1}{2}e^{(-x^2)}+C$