**Subject for this video:**

**Consumers' Surplus**

**Reading:**

- **General:** Section 6.2 Applications in Business and Economics
- **More Specifically:** bottom of page 403 – middle of page 405 Example 5

**Homework:**

H88: Consumers' Surplus (6.2#69)

## Recall Business Terminology that we used earlier in the semester

**Business Terminology from earlier in the semester**

**Demand**, $x$ (small letter), is a variable that represents the number of items made and sold.

**Price**, $p$ (small letter), is a variable that represents the selling price per item.

For what we will be discussing in the next three videos, it will be useful to slightly change the terminology, and to be more specific.

**Revised Business Terminology for Chapter 6**

**Quantity Demanded**, $x$ or $q$ (for *quantity*), is a variable that represents the number of items that consumers are willing to buy.

**Demand Price**, $p = D(x)$ or $p = D(q)$ The letter $p$ (small $p$), is a variable that represents the selling price that is necessary for consumers to be willing to buy the quantity $x$ or $q$. The value of the variable $p$ is given by a function $D(x)$ or $D(q)$ called the *Demand Price Function*. The *graph* of the Demand Price Function is called the *Demand Price Curve*.

Note that when the selling price is *high*, consumers will not be willing to buy many of the items. But when the selling price is *low*, consumers will be willing to buy a lot of the item. Therefore, the Demand Price curve $p = D(x)$ will go down as one moves from left to right. That is, the Demand Price function $p = D(x)$ will be a *decreasing* function.

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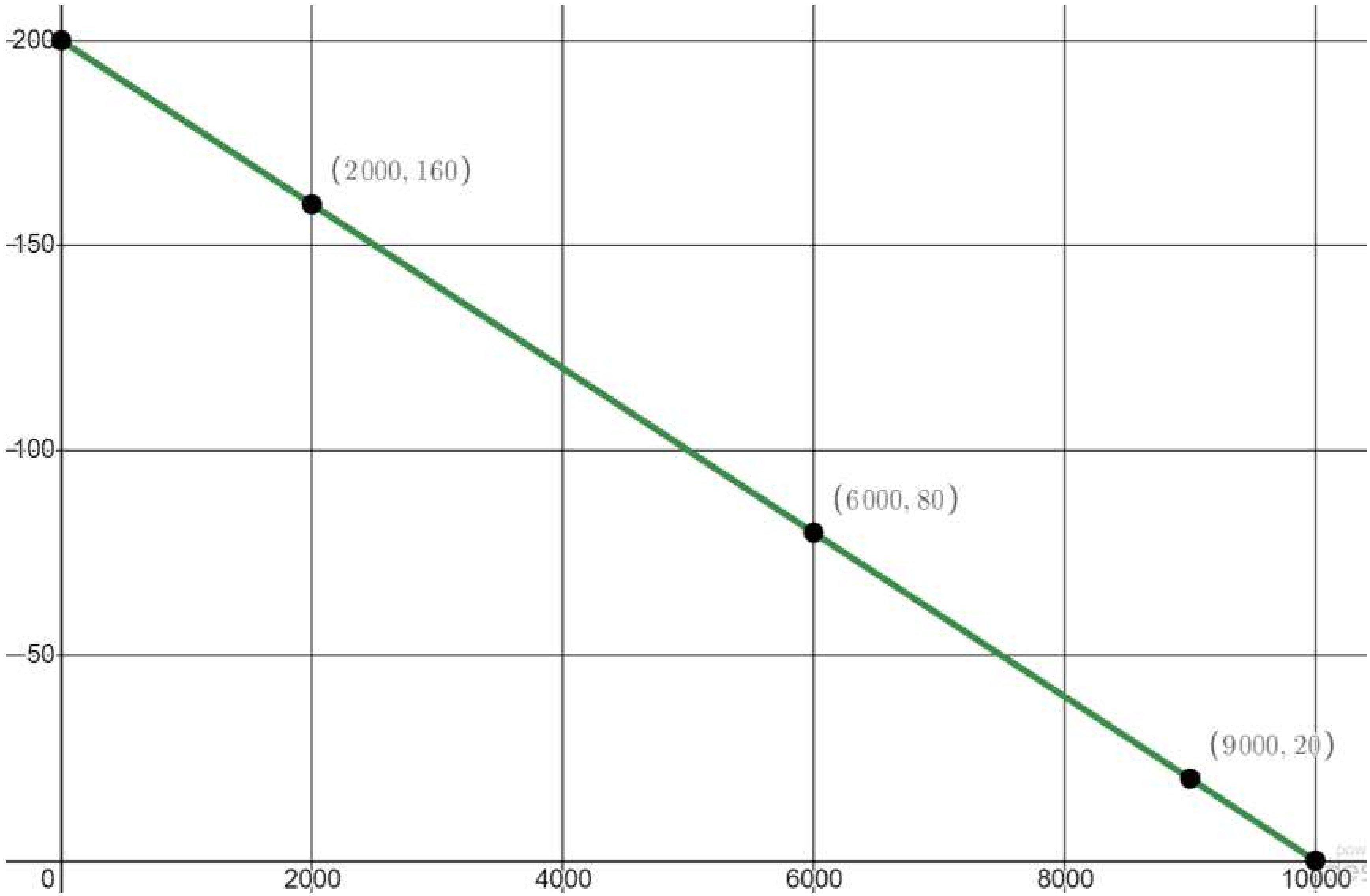
For example, a Demand Price Function could be $p = D(x) = 200 - 0.02x$



**Consumers' Surplus**

Suppose that the Demand Price Function is the one we discussed above, $p = D(x) = 200 - 0.02x$. And suppose that the selling price of the item has been established at $p = 80$. We see that on the $D(x)$ curve, the corresponding quantity is $x = 6000$. We could denote this quantity and price by $\bar{x}$ and $\bar{p}$. That is, $(\bar{x}, \bar{p}) = (6000{,}80)$.

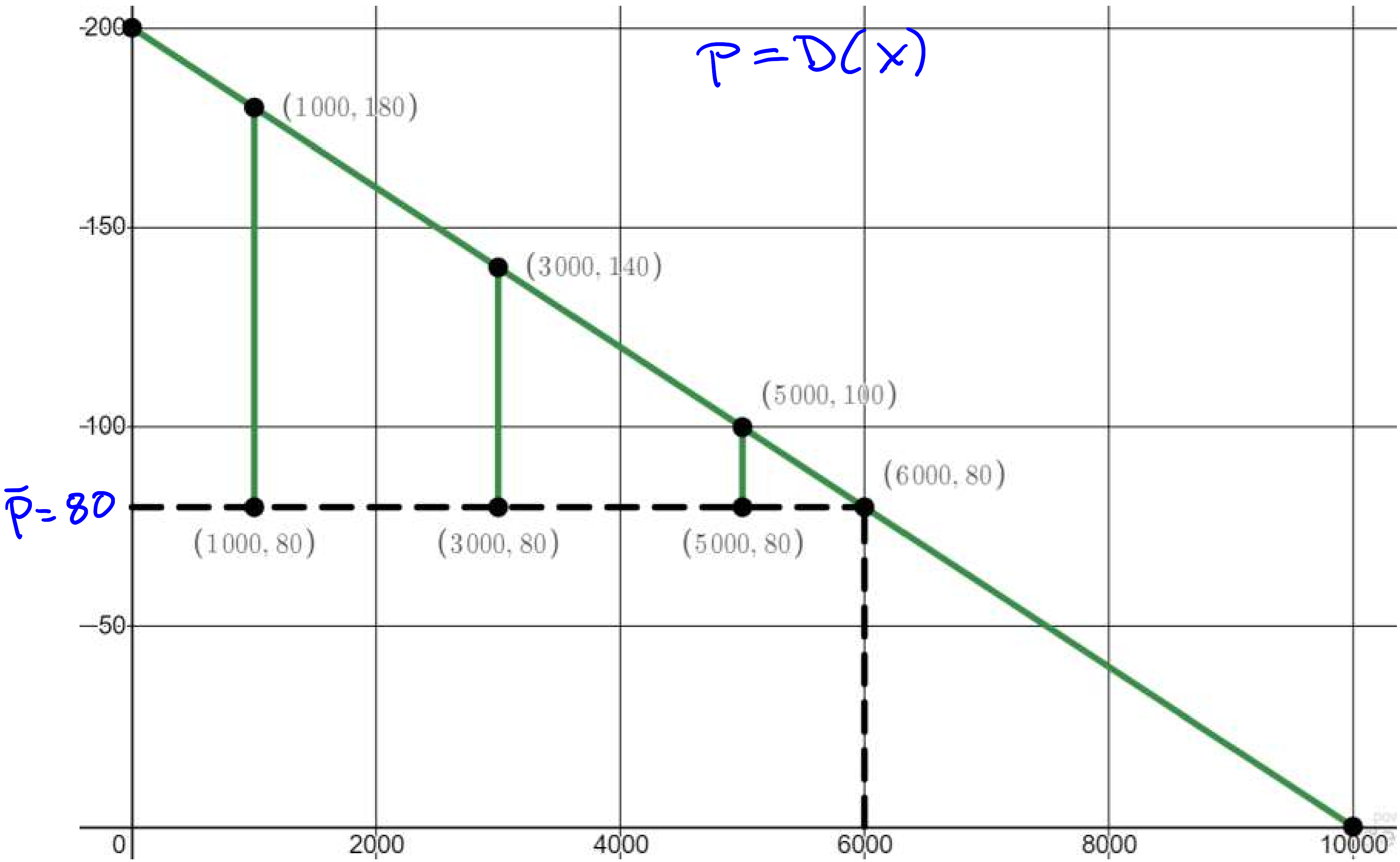
Consider who would buy the item at this price:

- Any consumer who would have been willing to pay more than \$80 for the item will buy the item if it is selling for \$80.

And consider how much those consumers will feel like they saved.

- Those who were willing to pay \$180 will feel like they saved \$100.
- Those who were willing to pay \$140 will feel like they saved \$60.
- Those who were willing to pay \$100 will feel like they saved \$20.

The amounts that these consumers will feel like they saved can be illustrated on the graph of the Demand Price Function.

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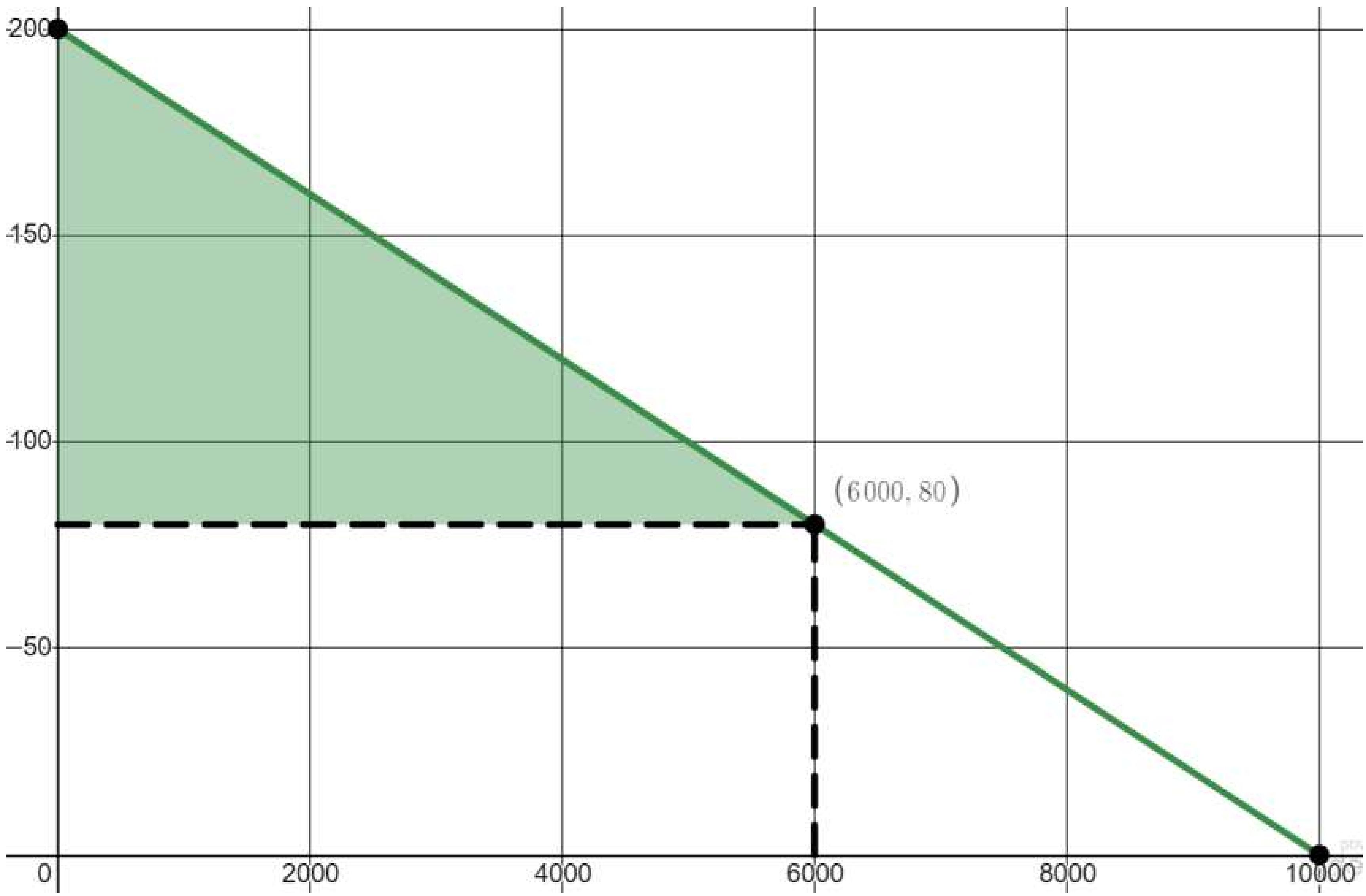
We see that the area of the region between the Demand Price Curve $D(x)$ and the horizontal line $p = \bar{p} = 80$ from $x = 0$ to $x = \bar{x} = 6000$ will correspond to the total amount that all consumers who buy the item will feel like they saved if the selling price is \$80.



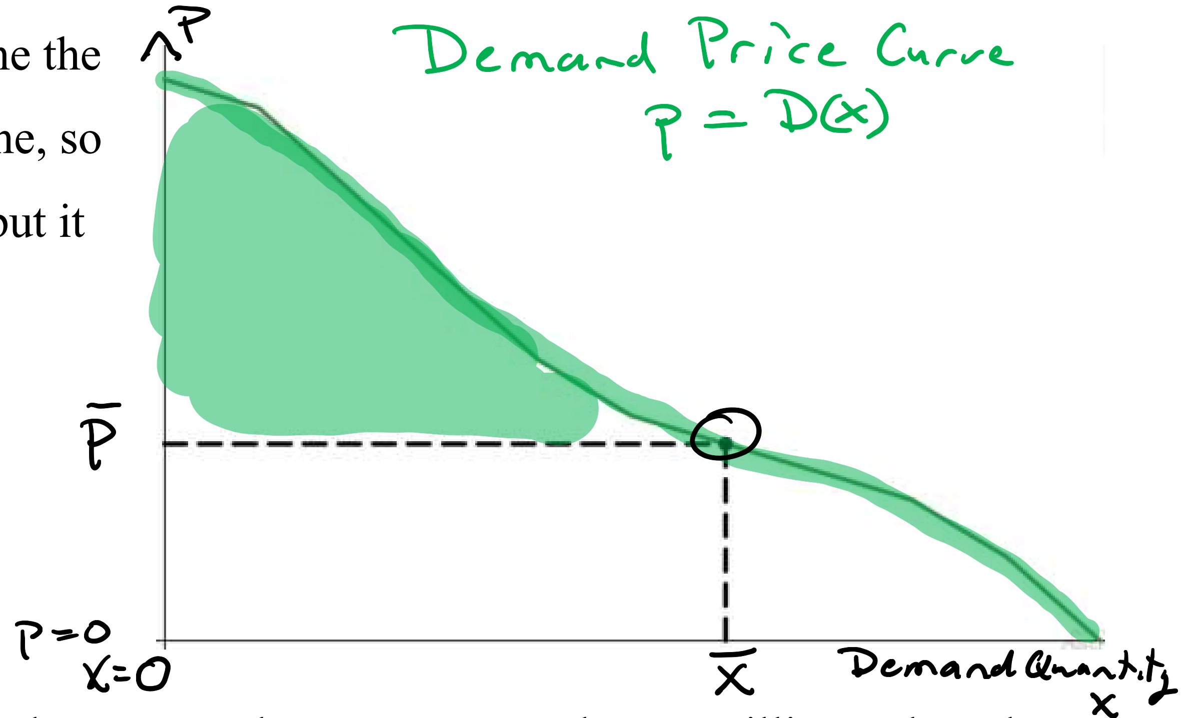
This region is just a triangle. We can find its area easily using geometry.

$$A = \frac{1}{2} b \cdot h = \frac{1}{2} \cdot 6000 \cdot 120 = 360{,}000$$

So, the total amount that all consumers will feel like they have saved is \$360,000.

This idea can be generalized. In general, the the Demand Price Curve $D(x)$ will not be a line, so the region will not be a triangular region, but it will be a simple region with

- top curve: $top(x) = D(x)$
- bottom curve: $bottom(x) = \bar{p}$
- left endpoint: $x = 0$
- right endpoint: $x = \bar{x}$.




So the area of the region, corresponding to the amount that consumers who are willing to buy the item at the price $\bar{p}$ will feel like they saved if the selling price is $\bar{p}$, will be the result of a definite integral computing the area between curves.

The resulting number, the area between curves, is called the ***Consumer's Surplus*** for the Demand Price Curve $D(x)$ and the price point $(\bar{x}, \bar{p})$.

The definition follows on the next page.

**Definition of Consumers' Surplus**

**Words: Consumer's Surplus** for the Demand Price Function $D(x)$ at the price point $(\bar{x}, \bar{p})$

**Usage:** $(\bar{x}, \bar{p})$ is a point on the the Demand Price curve $D(x)$

**Meaning in symbols:** the value of this definite integral:

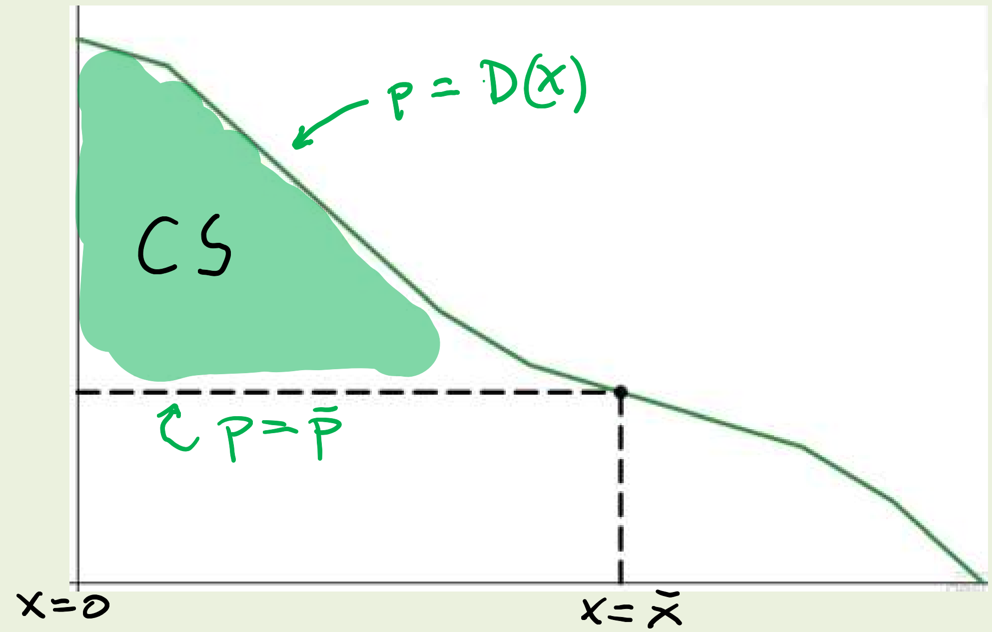
$$CS = \int_0^{\bar{x}} [D(x) - \bar{p}]dx$$

**Meaning in words:** $CS$ is the total amount that all consumers who are willing to buy the item at the price $\bar{p}$ will feel like they saved if the selling price is $\bar{p}$.

**Graphical Interpretation:**

$CS$ is the area of the simple region with

- top curve: $top(x) = D(x)$
- bottom curve: $bottom(x) = \bar{p}$
- left endpoint: $x = 0$
- right endpoint: $x = \bar{x}$.



**[Example 1]** Suppose that the *Demand Price Function* is $p = D(x) = 30 - 2x$ and the selling price has been established as $\bar{p} = 18$.

**(a)** Find the *Consumers' Surplus*.

**Solution:**

We must compute the definite integral $CS = \int_0^{\bar{x}} [D(x) - \bar{p}]dx$

In order to do this, we must first find the quantity $\bar{x}$ corresponding to the price $\bar{p} = 18$.

To do that, we will set $p = 18$ in the equation $p = 30 - 2x$ and solve for $x$.

The result is, $x = 6$. Therefore, $\bar{x} = 6$

The calculation of the Consumers' Surplus follows on the next page

Consumers' Surplus Calculation

$$CS = \int_0^{\bar{x}} [D(x) - \bar{p}]dx$$

$$= \int_0^6 [\underbrace{(30 - 2x) - 18}_{\text{simplify the integrand before integrating!}}]dx$$

$$= \int_0^6 12 - 2xdx$$

$$\underset{FTC}{=} \left(\int 12 - 2xdx\right)\Big|_0^6$$

$$= (12x - x^2 + C)|_0^6$$

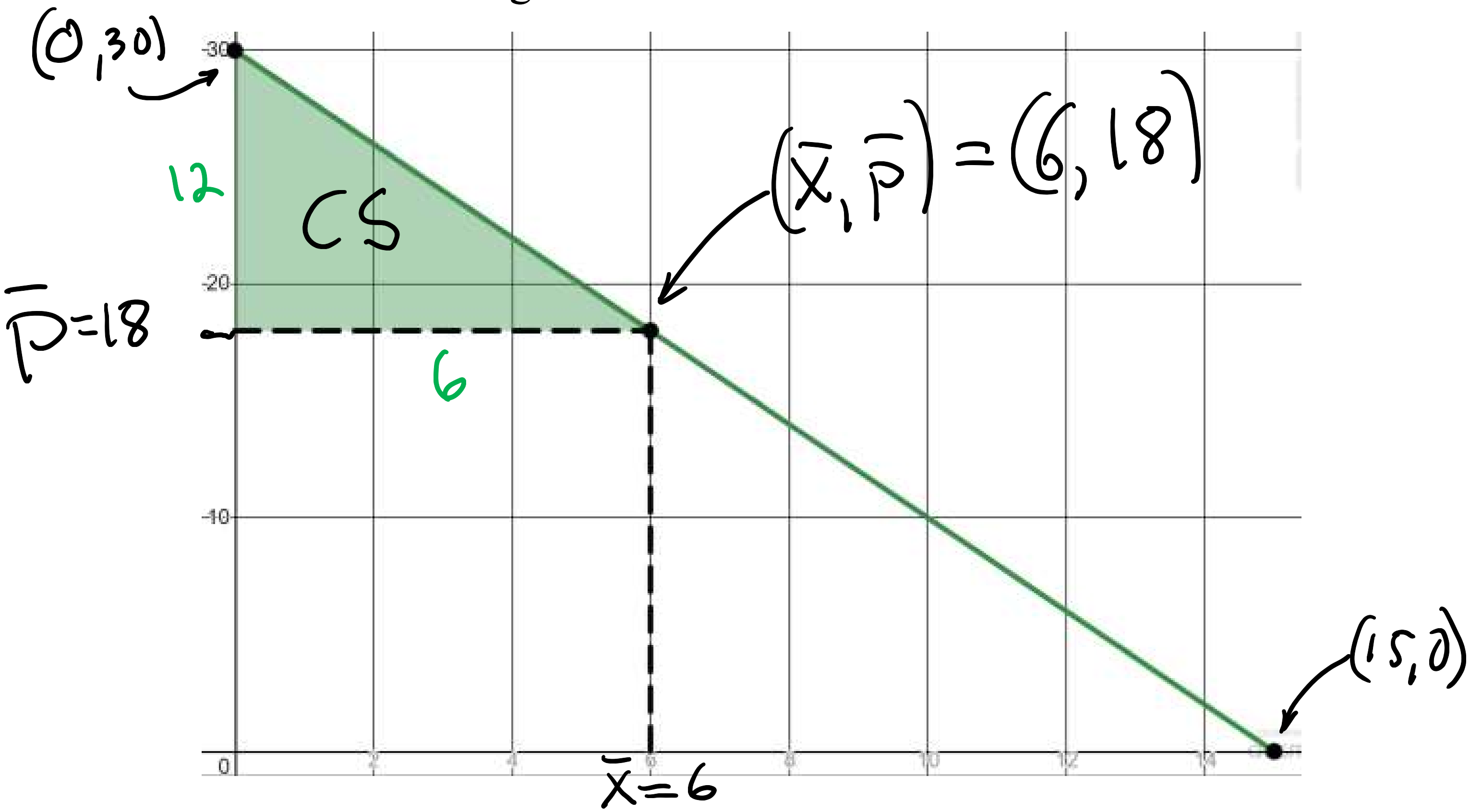
$$= (12(6) - (6)^2 + C) - (12(0) - (0)^2 + C)$$

$$= 72 - 36$$

$$= 36$$

**(b)** Illustrate with a drawing.

**Solution:** The Demand Price curve $p = D(x) = 30 - 2x$ will be a line with $y$ intercept at $(0,30)$ and slope $m = -2$. Therefore, its $x$ intercept will be at $(15,0)$. The price point $(\bar{x}, \bar{p}) = (6,18)$ will be a point on this line. The shaded region shown has $Area = CS = 36$.




Observe that this region is just a triangle. We can also find its area easily using geometry.

$$A = \frac{1}{2} b \cdot h = \frac{1}{2} \cdot 6 \cdot 12 = 36$$

**End of [Example 1]**