Subject for this video:

Producers' Surplus

Reading:

- **General:** Section 6.2 Applications in Business and Economics
- **More Specifically:** middle of page 405 – middle of page 406 Example 6

Homework:

H89: Producers' Surplus (6.2#74)

Previous terminology and concepts that will be useful in this video

Recall Business Terminology that we used earlier in the semester

Business Terminology from earlier in the semester

Demand, x (small letter), is a variable that represents the number of items made and sold. This sounds simple enough, but there can be complications. For example, in some problems, x represents the number of thousands of items made.

Price, p (small letter), is a variable that represents the selling price per item.

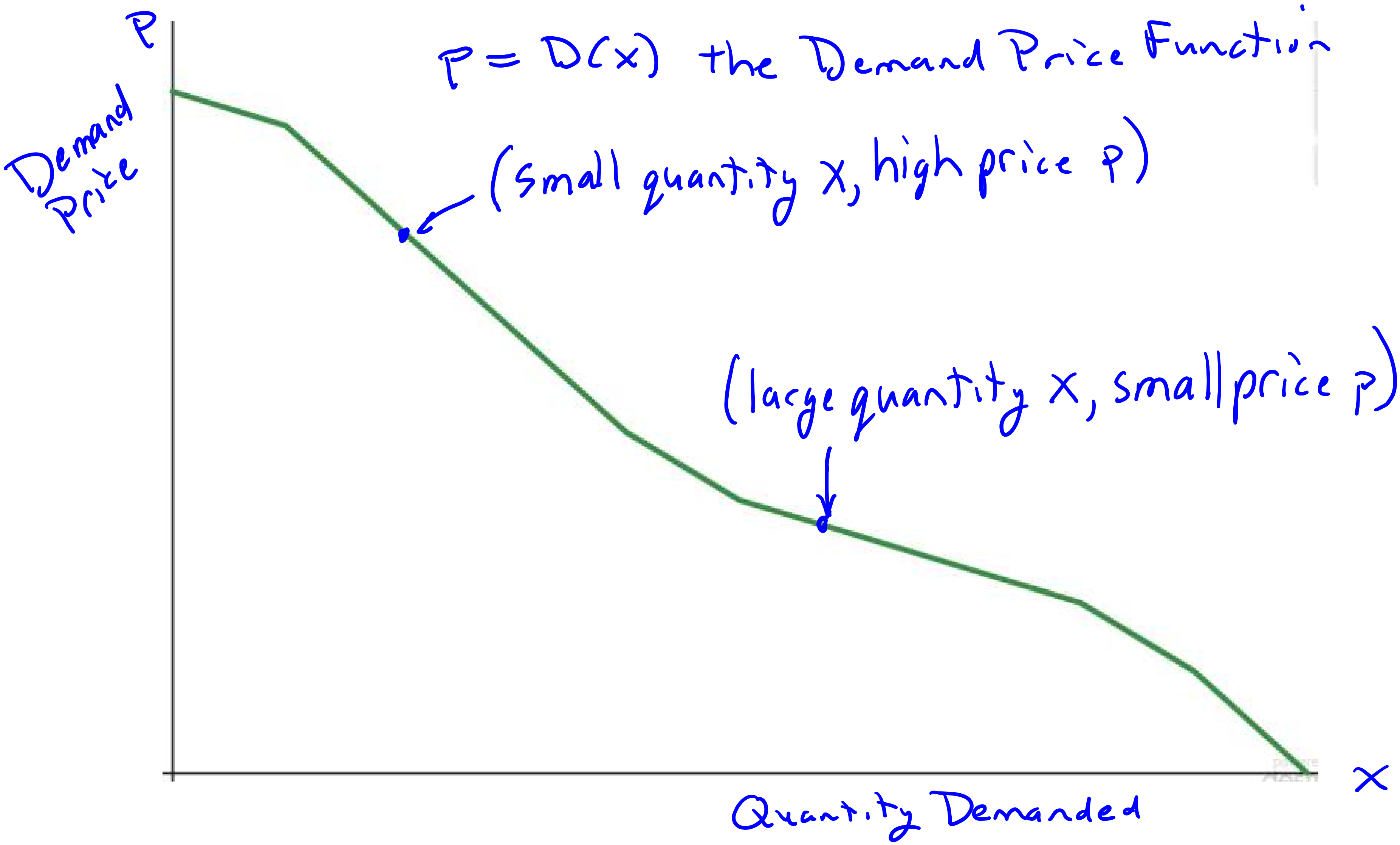
And recall that for Chapter 6, it is useful to slightly change the terminology, to be more specific:

Revised Business Terminology for Chapter 6

Quantity Demanded, x or q (for *quantity*), is a variable that represents the number of items that consumers are willing to buy.

Demand Price, $p = D(x)$ or $p = D(q)$ The letter p (small p), is a variable that represents the selling price that is necessary for consumers to be willing to buy the quantity x or q. The value of the variable p is given by a function $D(x)$ or $D(q)$ called the *Demand Price Function*. The *graph* of the Demand Price Function is called the *Demand Price Curve*.

And recall that when the selling price is *high*, consumers will not be willing to buy many of the items. But when the selling price is *low*, consumers will be willing to buy a lot of the item. Therefore, the Demand Price curve $p = D(x)$ will go down as one moves from left to right. That is, the Demand Price Function $p = D(x)$ will be a *decreasing* function.



Finally, recall the definition of Consumers' Surplus

Definition of Consumers' Surplus

Words: Consumers' Surplus for the Demand Price Function $D(x)$ at the price point $(\bar{x}, \bar{p})$

Usage: $(\bar{x}, \bar{p})$ is a point on the the Demand Price curve $D(x)$

Meaning in symbols: the value of this definite integral:

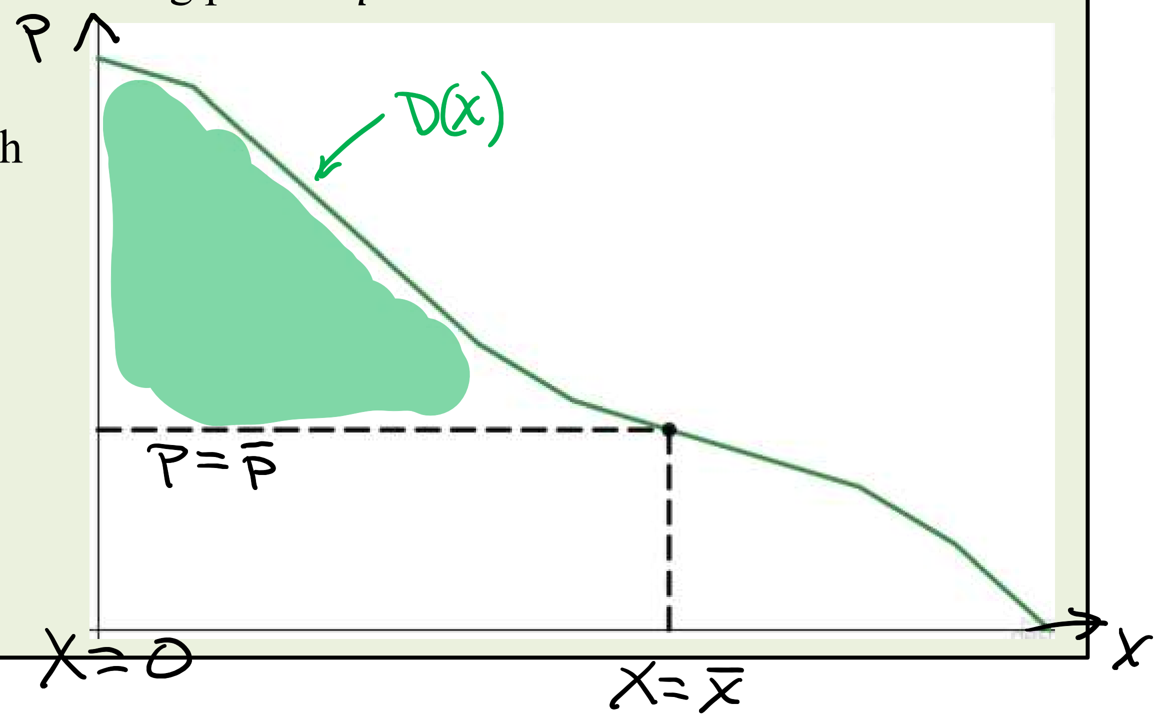
$$CS = \int_0^{\bar{x}} [D(x) - \bar{p}]dx$$

Meaning in words: CS is the total amount that all consumers who are willing to buy the item at the price $\bar{p}$ will feel like they saved if the selling price is $\bar{p}$.

Graphical Interpretation:

CS is the area of the simple region with

- top curve: $top(x) = D(x)$
- bottom curve: $bottom(x) = \bar{p}$
- left endpoint: $x = 0$
- right endpoint: $x = \bar{x}$.



Supply Price

So far in this course, the discussion of selling price and quantity sold has always been considered from the standpoint of the *consumer*: When the selling price of an item is *high*, consumers will not be willing to buy many of the item. But when the selling price is *low*, consumers will be willing to buy a lot of the item. That is why the graph of the Demand Price curve $p = D(x)$ will go down as one moves from left to right. That is, the Demand Price curve $p = D(x)$ is a *decreasing* function.

Now consider the *producers* of the goods: Realize that when the selling price for an item is *low*, producers of the item will not be willing to supply many of the items. But when the selling price is *high*, producers will be willing to supply a lot of the item.

We need to introduce function that will describe this behavior of producers. That is the idea of the *Quantity Supplied* and the *Supply Price Function*.

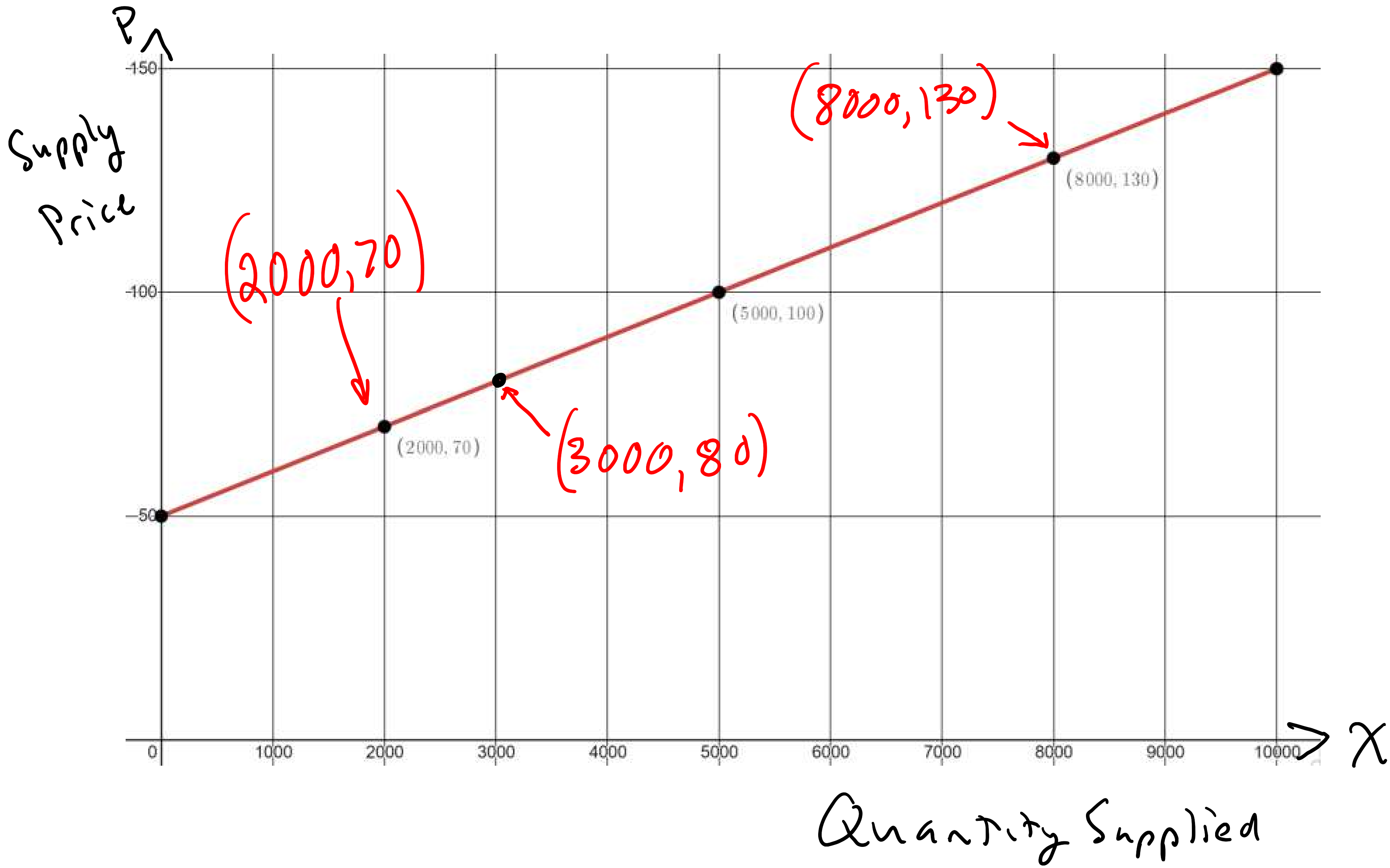
More New Business Terminology for Chapter 6

Quantity Supplied, x or q (for *quantity*), is a variable that represents the number of items that *producers* are willing to *supply*.

Supply Price, $p = S(x)$ or $p = S(q)$ The letter p (small p), is a variable that represents the selling price that is necessary for producers to be willing to supply the quantity x or q. The value of the variable p is given by a function $S(x)$ or $S(q)$ called the *Supply Price Function*. The *graph* of the Supply Price Function is called the *Supply Price Curve*.

In light of our earlier observation that when the selling price for an item is *low*, producers of the item will not be willing to supply many of the items, but when the selling price is *high*, producers will be willing to supply a lot of the item, we can conclude that the graph of of the Supply Price curve $p = S(x)$ will go up as one moves from left to right. That is, the Supply Price curve $p = S(x)$ will be an *increasing* function.

For example, a Supply Price Function could be $p = S(x) = 0.01x + 50$



Suppose that the Supply Price Function is the one we discussed above, $p = S(x) = 0.01x + 50$. And suppose that the selling price of the item has been established at $p = 80$. We see that on the $S(x)$ curve, the corresponding quantity is $x = 3000$. We could denote this quantity and price by $\bar{x}$ and $\bar{p}$. That is, $(\bar{x}, \bar{p}) = (3000,80)$.

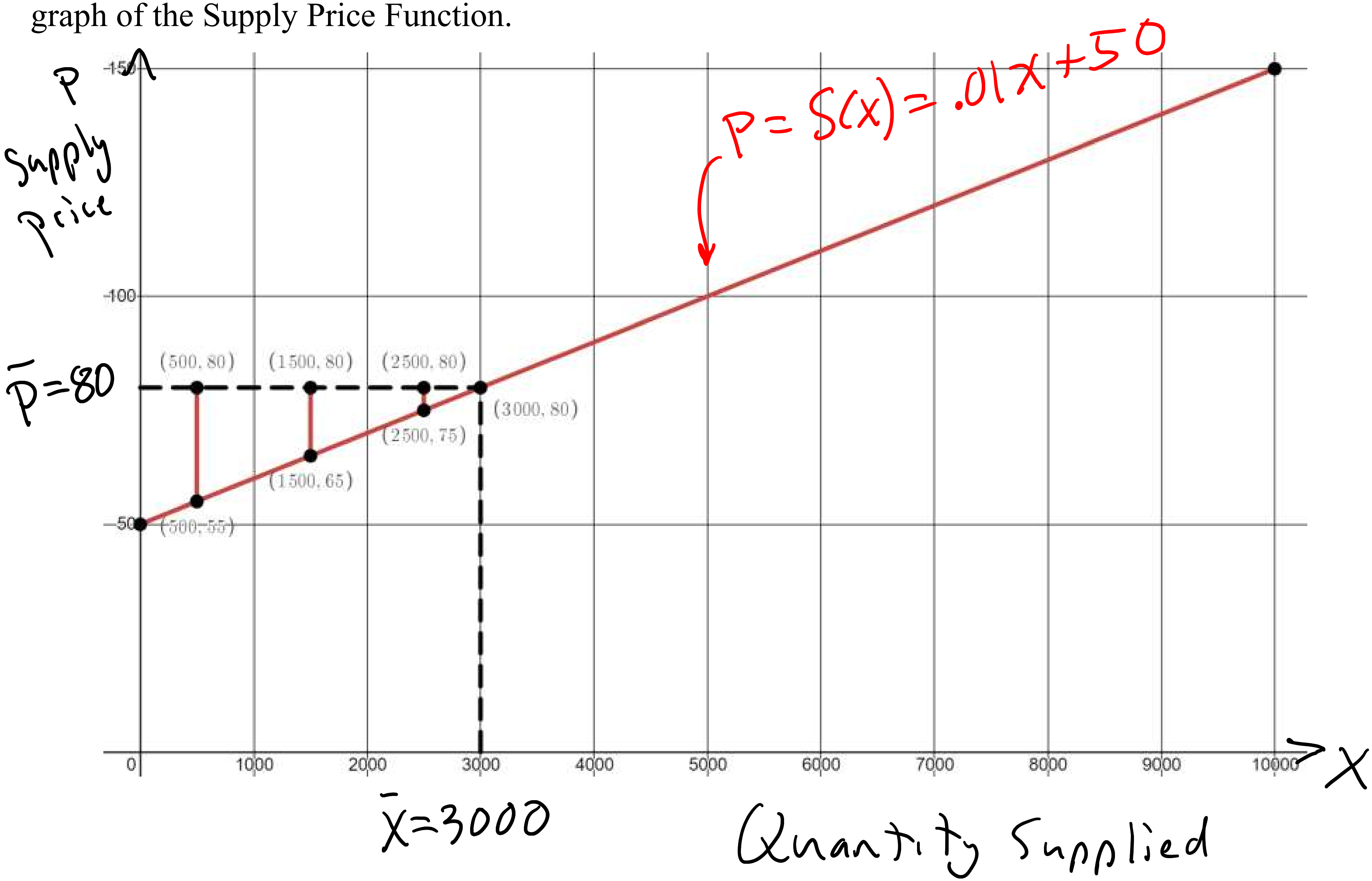
Consider who would supply the item at this price:

- Any producer who would have been willing to supply the item for less than \$80 will be willing to sell the item for \$80.

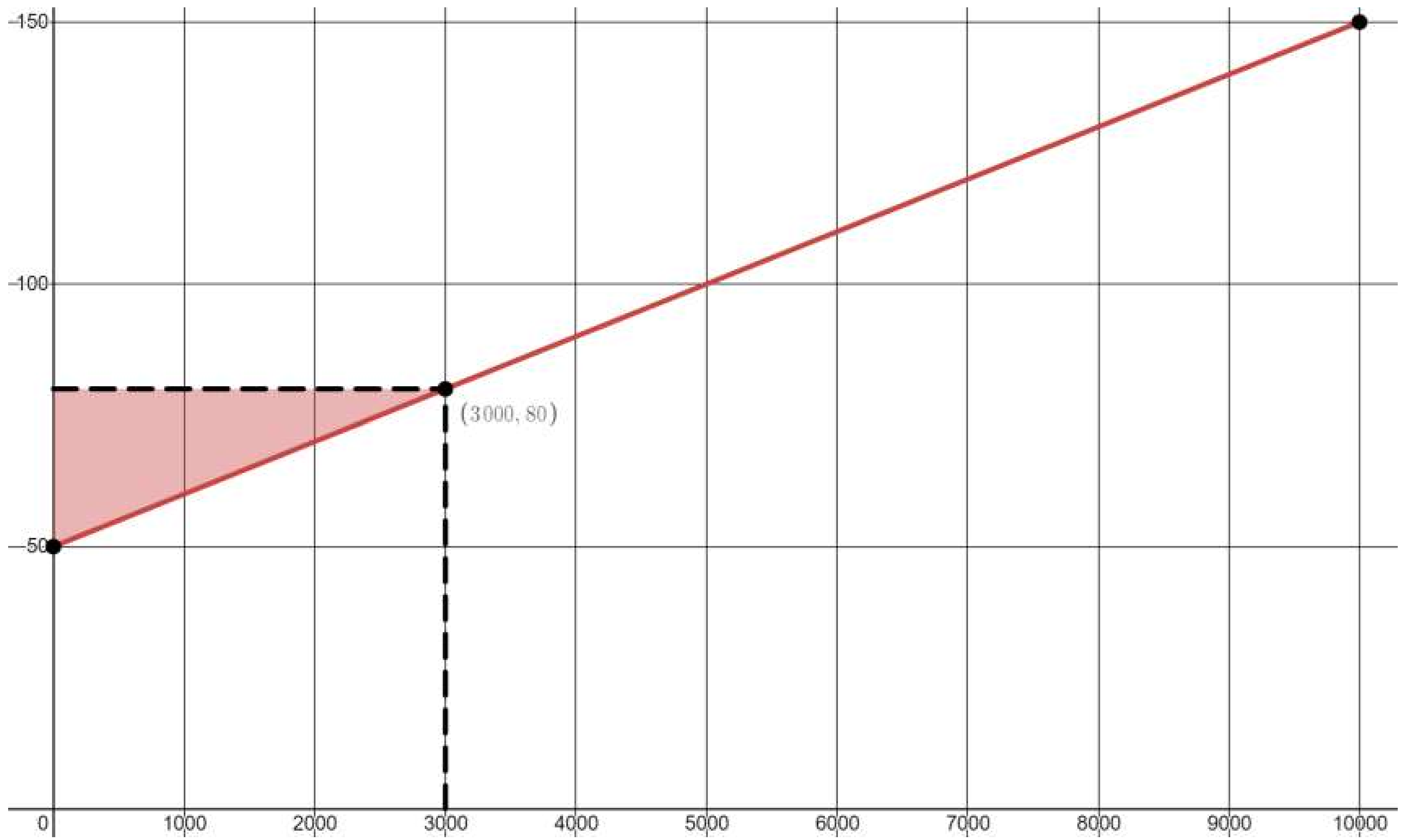
And consider how much extra money these producers will fell like they made.

- Those who were willing to sell for \$55 will feel like they made \$25 extra.
- Those who were willing to sell for \$65 will feel like they made \$15 extra.
- Those who were willing to sell for \$75 will feel like they made \$5 extra.

The amounts of extra money that these producers will feel like they made can be illustrated on the graph of the Supply Price Function.



We see that the area between the Supply Curve $S(x)$ and the horizontal line $p = \bar{p} = 80$ from $\overset{X}{\cancel{q}} = 0$ to $x = \bar{x} = 3000$ will correspond to the total amount of extra money that all producers who sell the item will feel like they made if the selling price is $80.



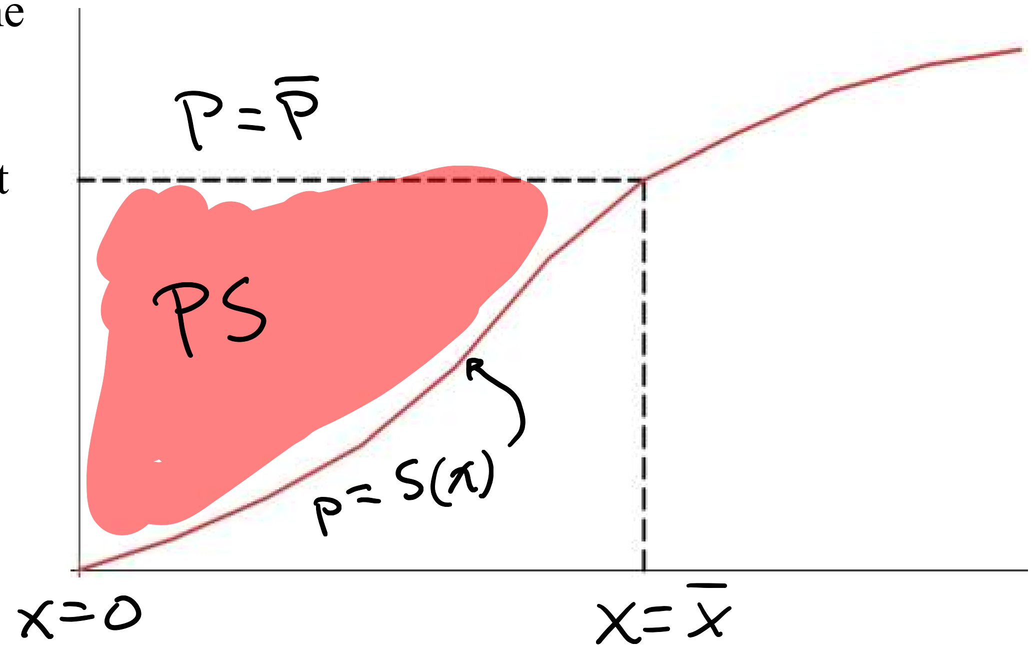
This region is just a triangle. We can find its area easily using geometry.

$$A = \frac{1}{2} b \cdot h = \frac{1}{2} \cdot 3000 \cdot 30 = 45{,}000$$

So the total amount of extra money that all producers will feel like they made is $45,000.

This idea can be generalized. In general, the the Supply Price Curve $S(x)$ will not be a line, so the region will not be a triangular region, but it will be a simple region with

- top curve: $top(x) = \bar{p}$
- bottom curve: $bottom(x) = S(x)$
- left endpoint: $x = 0$
- right endpoint: $x = \bar{x}$.



So the area of the region, corresponding to the amount that producers who are willing to supply the item at the price $\bar{p}$ will feel like they made extra if the selling price is $\bar{p}$, will be the result of a definite integral computing the area between curves.

The resulting number, the area between curves, is called the ***Producers's Surplus*** for the Supply Price Function $S(x)$ at the price point $(\bar{x}, \bar{p})$.

The definition follows on the next page.

Definition of Producers' Surplus

Words: Producers' Surplus for the Supply Price Function $S(x)$ at the price point $(\bar{x}, \bar{p})$

Usage: $(\bar{x}, \bar{p})$ is a point on the the Supply Price curve $S(x)$

Meaning in symbols: the value of this definite integral:

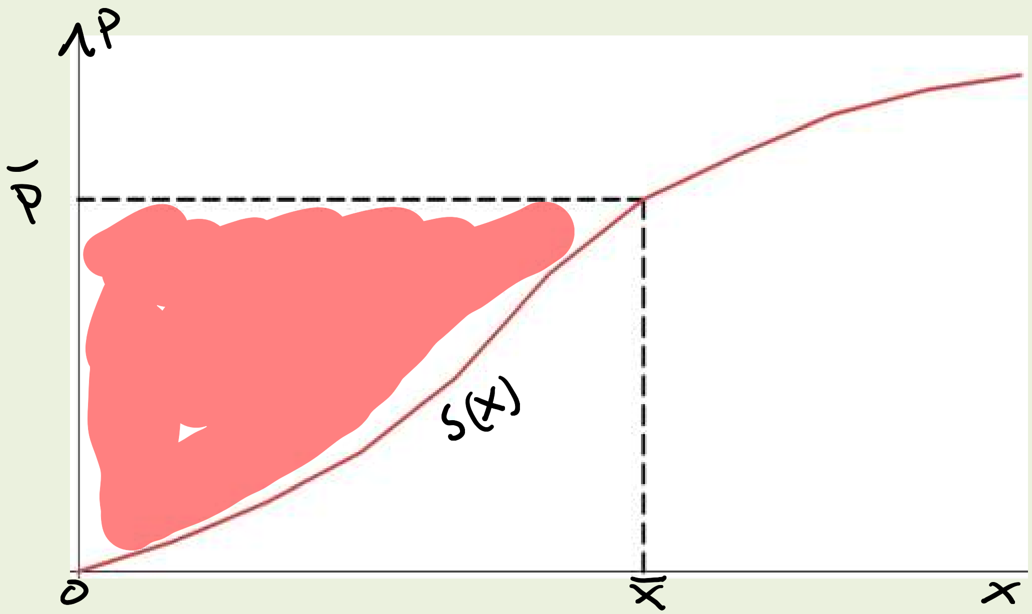
$$PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx$$

Meaning in words: PS is the total amount of extra money that all producers who are willing to supply the item at the price $\bar{p}$ will feel like they made if the selling price is $\bar{p}$.

Graphical Interpretation:

PS is the area of the simple region with

- top curve: $top(x) = \bar{p}$
- bottom curve: $bottom(x) = S(x)$
- left endpoint: $x = 0$
- right endpoint: $x = \bar{x}$.



[Example 1]

Suppose that the *Supply Price Function* is $p = S(x) = 12 + x$ and the selling price has been established as $\bar{p} = 18$.

(a) Find the Producers' Surplus.

Solution:

We must compute the definite integral $PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx$

In order to do this, we must first find the quantity $\bar{x}$ corresponding to the price $\bar{p} = 18$.

To do that, we will set $p = 18$ in the equation $p = 12 + x$ and solve for x.

The result is, $x = 6$. Therefore, $\bar{x} = 6$

The calculation of the definite integral follows on the next page

Producers' Surplus Calculation

$$PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx$$

$$= \int_0^6 [\underbrace{18 - (12 + x)}]dx$$

simplify the integrand before integrating

$$= \int_0^6 6 - xdx$$

$$\underset{FTC}{=} \left(\int 6 - xdx\right)\Big|_0^6$$

$$= \left(6x - \frac{x^2}{2} + C\right)\Big|_0^6$$

$$= \left(6(6) - \frac{(6)^2}{2} + C\right) - \left(6(0) - \frac{(0)^2}{2} + C\right)$$

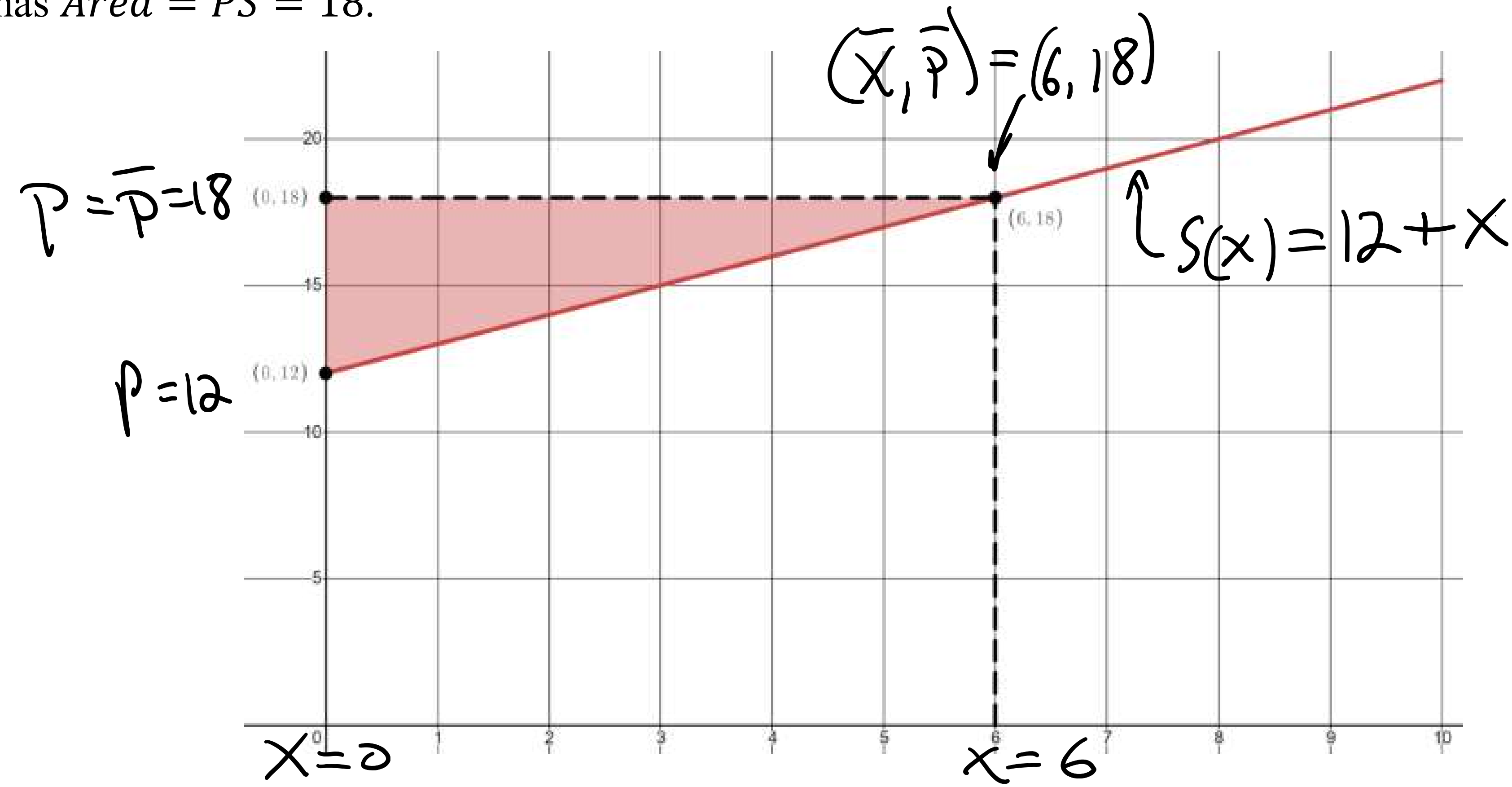
$$= 36 - 18$$

$$= 18$$

Producers' Surplus

(b) Illustrate with a drawing.

Solution: The *Supply Price Curve* $p = S(x) = 12 + x$ will be a line with y intercept at $(0,12)$ and slope $m = 1$. The price point $(\bar{x}, \bar{p}) = (6,18)$ will be a point on this line. The shaded region shown has $Area = PS = 18$.



Observe that this region is just a triangle. We can also find its area easily using geometry.

$$A = \frac{1}{2} b \cdot h = \frac{1}{2} \cdot 6 \cdot 6 = 18$$

End of [Example 1]

[Example 2]

(A) Find the Producers' Surplus at a price level of $\bar{p} = 80$ for the Supply Price Function

$$p = S(x) = 10 + .4x + 0.003x^2$$

Solution:

We must compute the definite integral $PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx$

In order to do this, we must first find the quantity $\bar{x}$ corresponding to the price $\bar{p} = 80$.

To do that, we will set $p = 80$ in the equation $p = 10 + .4x + 0.003x^2$ and solve for x.

$$10 + .4x + 0.003x^2 = 80$$

$$.003x^2 + .4x - 70 = 0$$

We can solve this equation using the quadratic formula. I will do that on the next page.

$a = .003$

$b = .4$

$c = -70$

Using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(.4) \pm \sqrt{(.4)^2 - 4(.003)(-70)}}{2(.003)}$$

$$= \frac{-0.4 \pm \sqrt{.16 + .84}}{0.006}$$

$$= \frac{-0.4 \pm \sqrt{1}}{0.006}$$

$$= \frac{-0.4 \pm 1}{0.006}$$

We are only interested in positive x values, so our solution must be

$$x = \frac{-0.4 + 1}{0.006} = \frac{0.6}{0.006} = 100$$

Therefore, $\bar{x} = 100$

The calculation of the Producers' Surplus follows on the next page.

Producers' Surplus Calculation

$$PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx$$

$$= \int_0^{100} \underbrace{[80 - (10 + .4x + 0.003x^2)]}dx$$

simplify the integrand before integrating

$$= \int_0^{100} 70 - .4x - .003x^2 dx$$

$$= \left(\int 70 - .4x - .003x^2 dx\right)\Big|_0^{100}$$

$$= (70x - .2x^2 - .001x^3 + C)|_0^{100}$$

$$= (70(100) - .2(100)^2 - .001(100)^3 + C) - (70(0) - .2(0)^2 - .001(0)^3 + C)$$

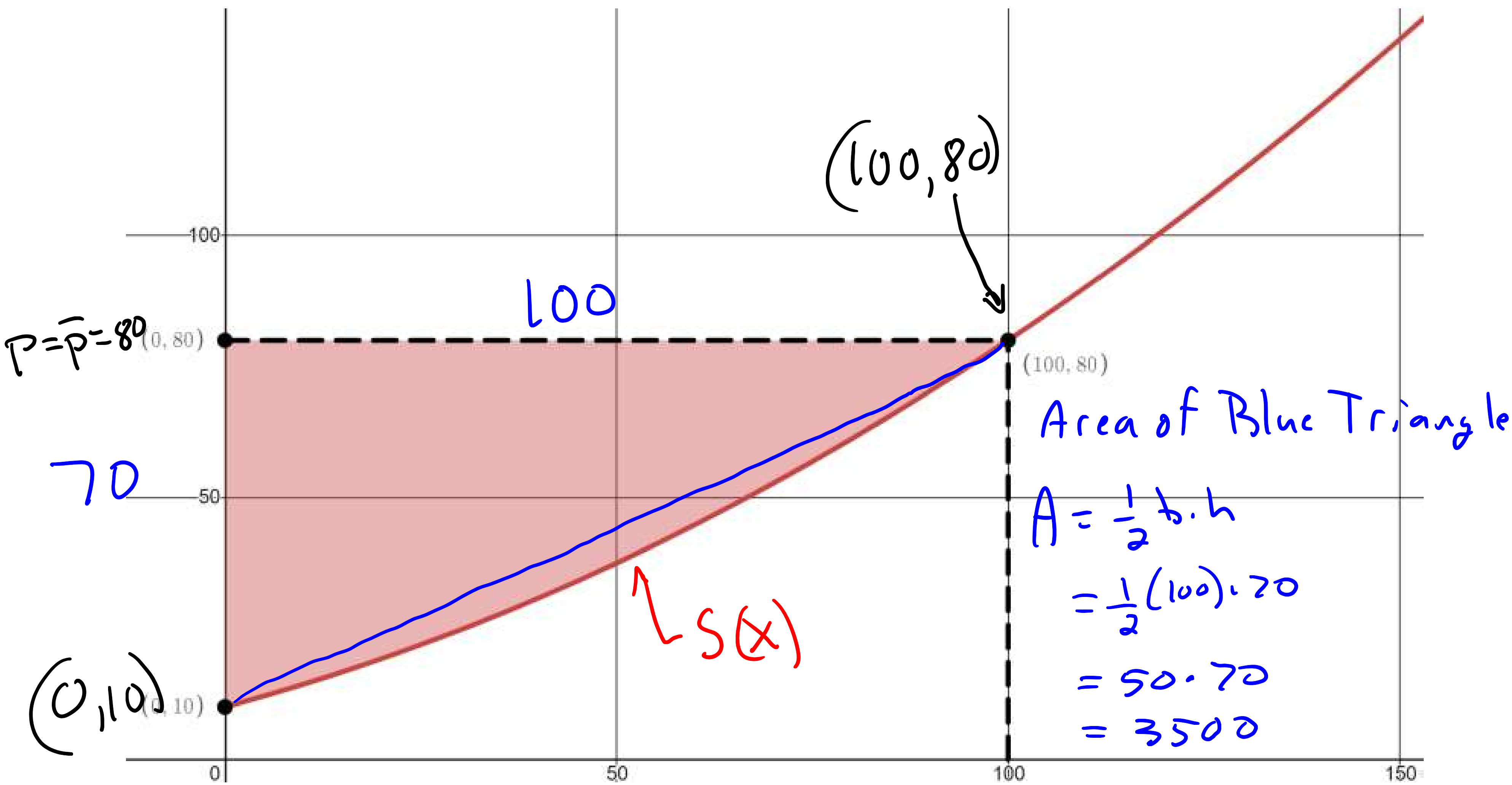
$$= 7000 - .2(10{,}000) - .001(1{,}000{,}000)$$

$$= 7000 - 2000 - 1000$$

$$= 4000$$

(b) Illustrate with a drawing.

Solution: The *Supply Price curve* $p = S(x) = p = S(x) = 10 + .4x + 0.003x^2$ will be an upward-facing parabola with y intercept at $(0,10)$. The price point $(\bar{x}, \bar{p}) = (100,80)$ will be a point on this line. The shaded region shown has $Area = PS = 4000$.



End of [Example 2]