Subject for this video:

Equilibrium Price Point and Consumers' Surplus and Producers' Surplus

Reading:

- **General:** Section 6.2 Applications in Business and Economics
- **More Specifically:** middle of page 406 – middle of page 407 Example 7

Homework: H90: Equilibrium Price Point

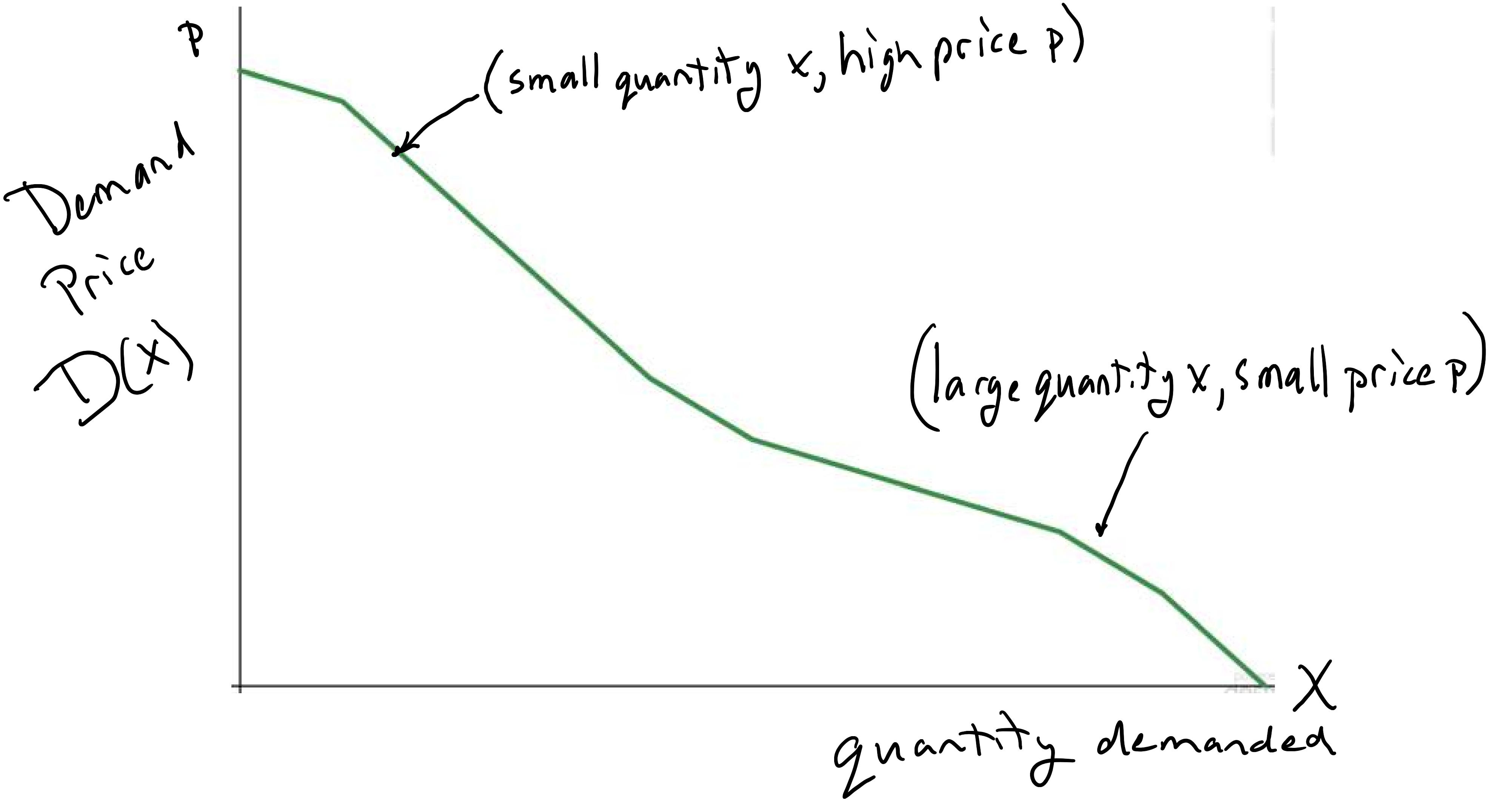
- Barnett 6.2#77,78*
- Lial/Greenwell/Ritchey Business Econ 7.5#35

Recall the concept of Quantity Demanded and the Demand Price from the Video for H88:

Quantity Demanded, x or q (for *quantity*), is a variable that represents the number of items that consumers are willing to buy.

Demand Price, $p = D(x)$ or $p = D(q)$ The letter p (small p), is a variable that represents the selling price that is necessary for consumers to be willing to buy the quantity x or q. The value of the variable p is given by a function $D(x)$ or $D(q)$ called the *Demand Price Function*. The *graph* of the Demand Price Function is called the *Demand Price Curve*.

And recall that when the selling price for an item is *high*, consumers will not be willing to buy many of the items, but when the selling price is *low*, consumers will be willing to buy a lot of the item. Therefore, the *Demand Price curve* will go *down* as one moves from left to right. That is, the *Demand Price Function* $p = D(x)$ will be a *decreasing* function.



Recall the definition of Consumers' Surplus

Definition of Consumers' Surplus

Words: Consumers' Surplus for the Demand Price Function $D(x)$ at the price point $(\bar{x}, \bar{p})$

Usage: $(\bar{x}, \bar{p})$ is a point on the the Demand Price curve $D(x)$

Meaning in symbols: the value of this definite integral:

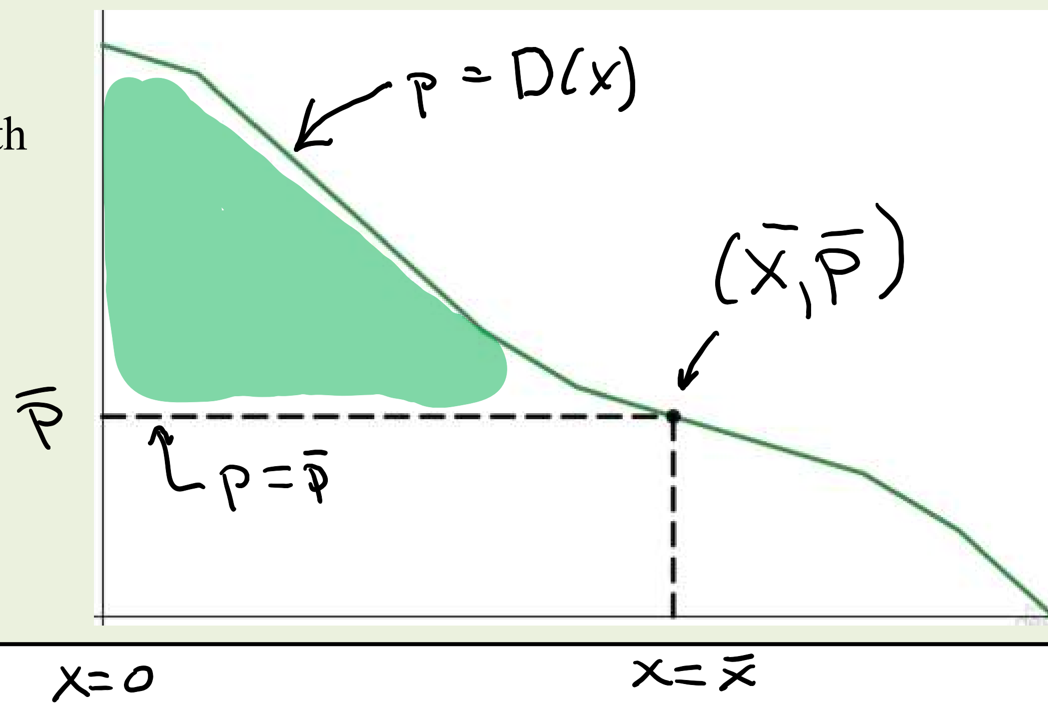
$$CS = \int_0^{\bar{x}} [D(x) - \bar{p}]dx$$

Meaning in words: CS is the total amount that all consumers who are willing to buy the item at the price $\bar{p}$ will feel like they saved if the selling price is $\bar{p}$.

Graphical Interpretation:

CS is the area of the simple region with

- top curve: $top(x) = D(x)$
- bottom curve: $bottom(x) = \bar{p}$
- left endpoint: $x = 0$
- right endpoint: $x = \bar{x}$.

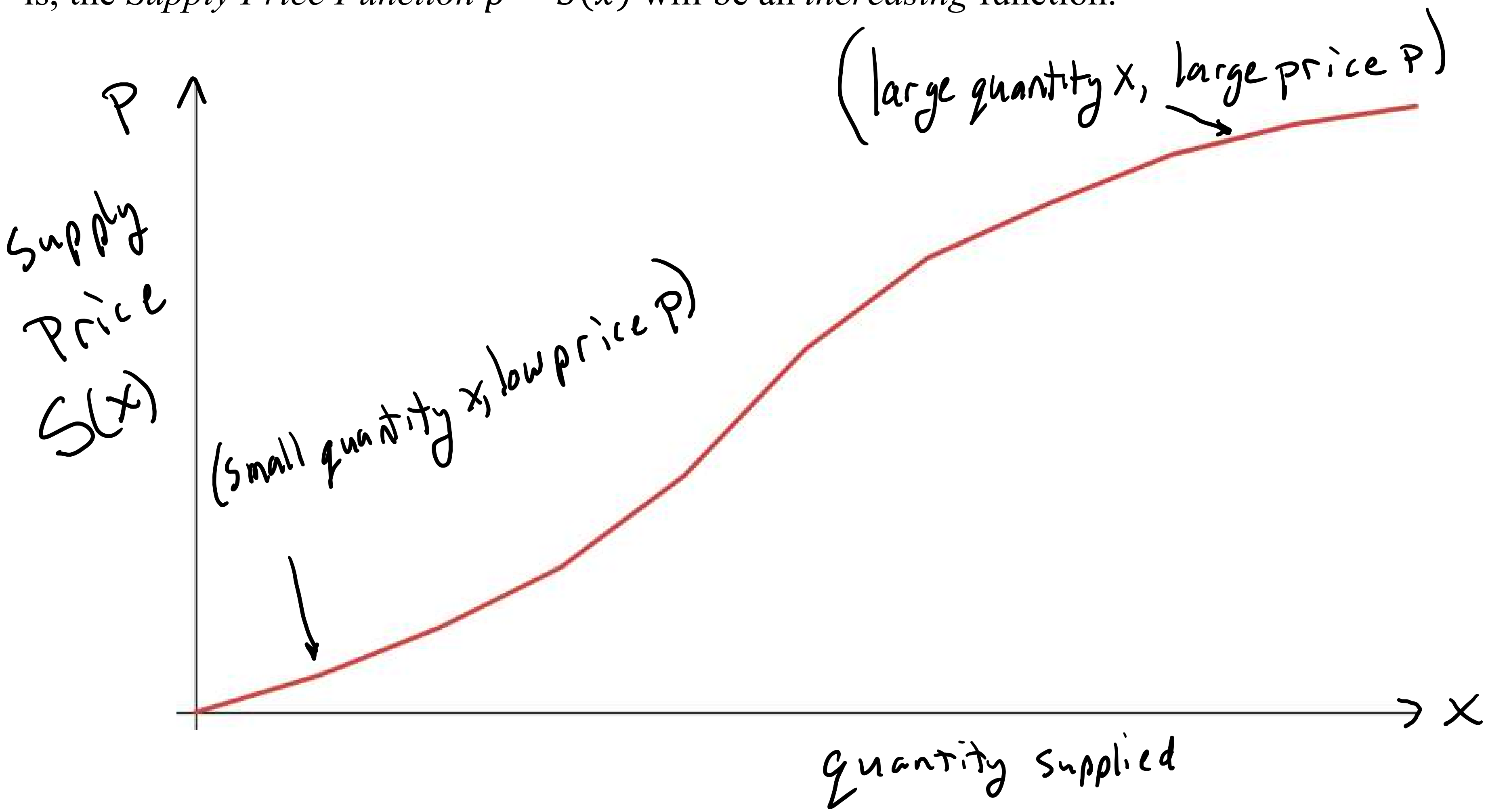


Recall the concept of Quantity Supplied and the Supply Price from the Video for H89:

Quantity Supplied, x or q (for *quantity*), is a variable that represents the number of items that *producers* are willing to *supply*.

Supply Price, $p = S(x)$ or $p = S(q)$ The letter p (small p), is a variable that represents the selling price that is necessary for producers to be willing to supply the quantity x or q. The value of the variable p is given by a function $S(x)$ or $S(q)$ called the *Supply Price Function*. The *graph* of the Supply Price Function is called the *Supply Price Curve*.

And recall that when the selling price for an item is *low*, producers of the item will not be willing to supply many of the items, but when the selling price is *high*, producers will be willing to supply a lot of the item. Therefore, the Supply Price curve will go *up* as one moves from left to right. That is, the *Supply Price Function* $p = S(x)$ will be an *increasing* function.



Recall the definition of Producers' Surplus

Definition of Producers' Surplus

Words: Producers' Surplus for the Supply Price Function $S(x)$ at the price point $(\bar{x}, \bar{p})$

Usage: $(\bar{x}, \bar{p})$ is a point on the the Supply Price curve $S(x)$

Meaning in symbols: the value of this definite integral:

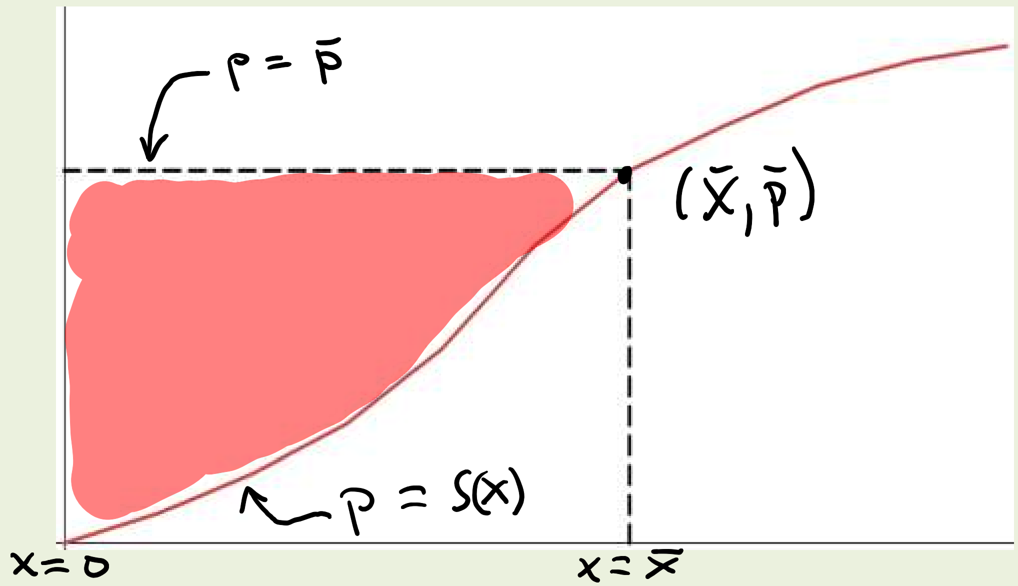
$$PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx$$

Meaning in words: PS is the total amount of extra money that all producers who are willing to supply the item at the price $\bar{p}$ will feel like they made if the selling price is $\bar{p}$.

Graphical Interpretation:

PS is the area of the simple region with

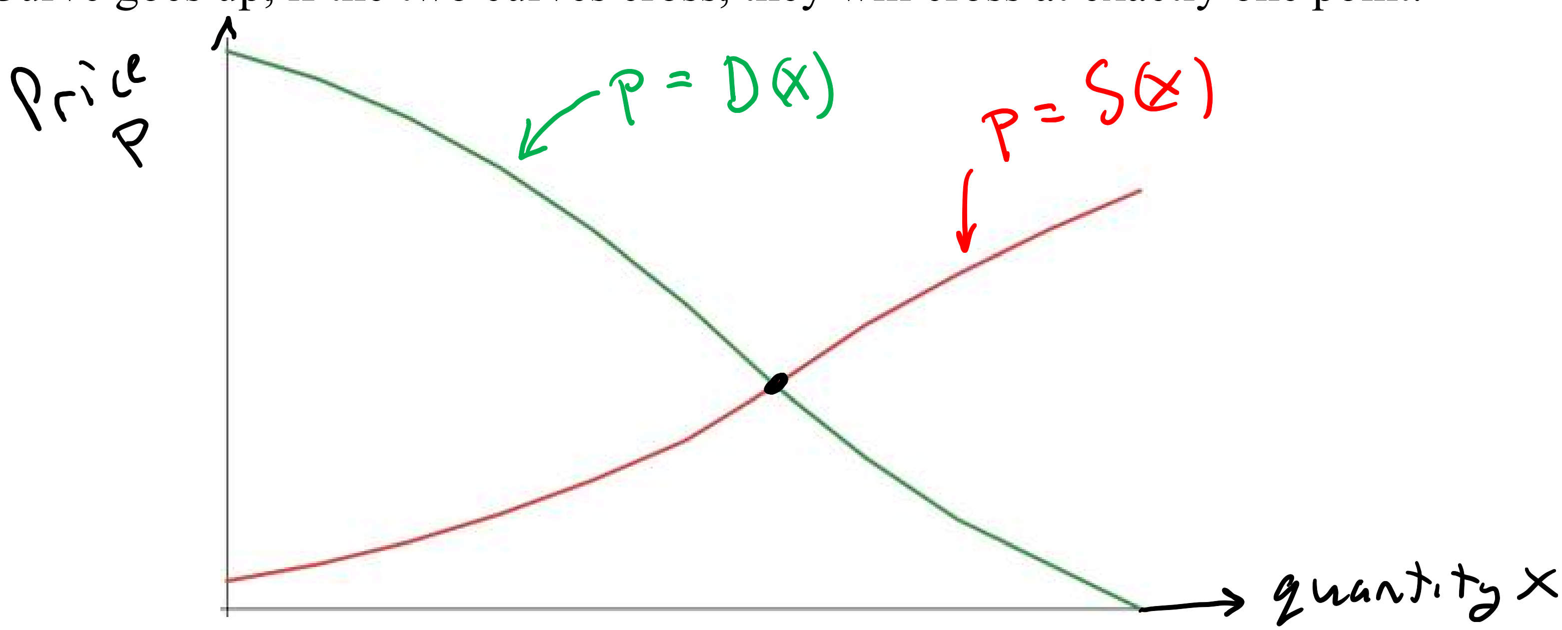
- top curve: $top(x) = \bar{p}$
- bottom curve: $bottom(x) = S(x)$
- left endpoint: $x = 0$
- right endpoint: $x = \bar{x}$.



Equilibrium Price and Quantity $(\bar{x}, \bar{p})$.

In the videos for Homeworks H88 and H89, we studied examples in which Consumers' Surplus or Producers' Surplus was calculated for a given price level $\bar{p}$.

In an actual competitive market, the price level for a particular item will be the price at which the quantity that producers are willing to supply equals the quantity that consumers are willing to buy. Observe that since the Demand Price Curve goes down as you move from left to right and the Supply Price Curve goes up, if the two curves cross, they will cross at exactly one point.



This point is called the *equilibrium price point*. The definition follows on the next page.

Definition of Equilibrium Quantity, Equilibrium Price, Equilibrium Point

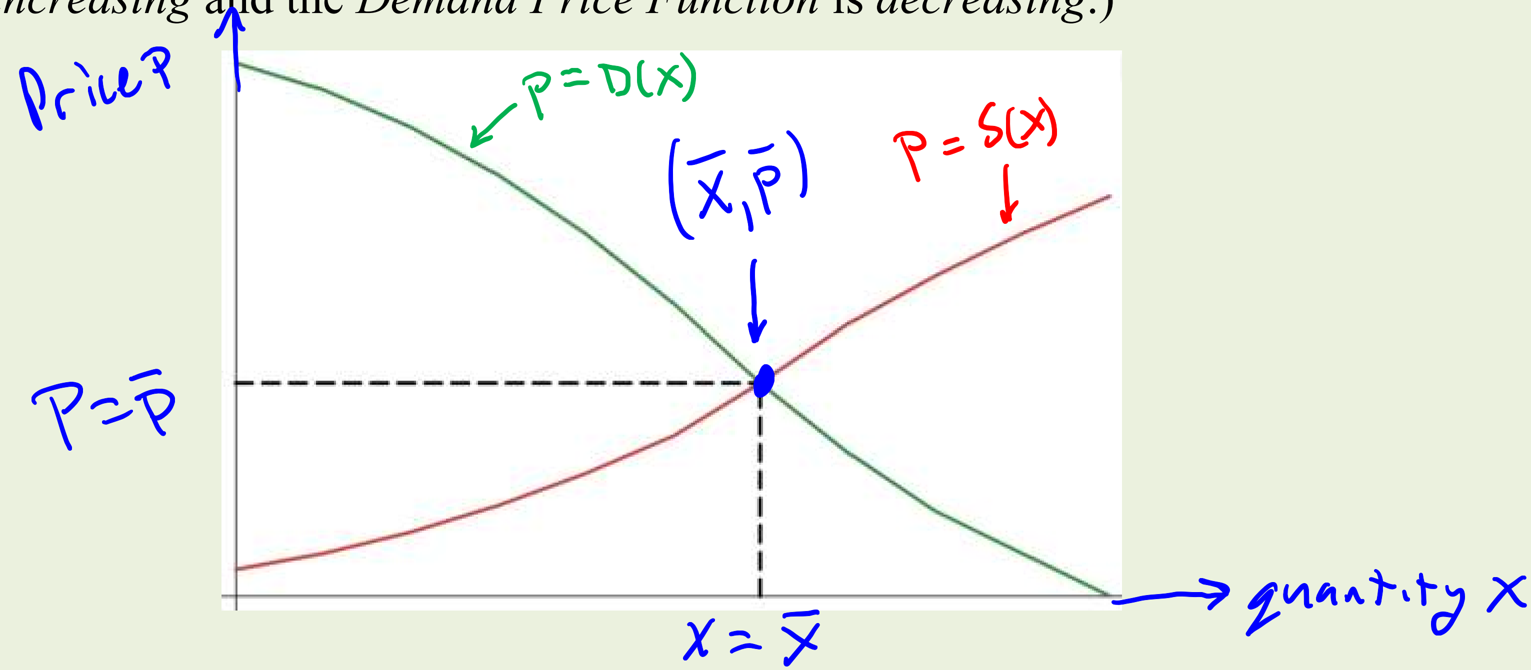
For a given *Supply Price Function* $p = S(x)$ and *Demand Price Function* $p = D(x)$,

the ***equilibrium quantity***, $\bar{\boldsymbol{x}}$, and ***equilibrium price***, $\bar{\mathbf{p}}$, are defined to be the quantity and price such that

$$S(\bar{x}) = \bar{p} = D(\bar{x})$$

The ***equilibrium price point*** is defined to be $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{p}})$.

Remark: There will be *at most one* equilibrium price point, because the *Supply Price Function* is *increasing* and the *Demand Price Function* is *decreasing*.)



[Example 1] (similar to 6.2#77)

Suppose that

- the *Demand Price Function* is $p = D(x) = 200 - 0.02x$
- the *Supply Price Function* is $p = S(x) = 0.01x + 50$

(a) Find the *equilibrium price point*.

Solution:

We set $D(x) = S(x)$ and solve for x.

$$200 - 0.02x = 0.01x + 50$$

$$150 = 0.03x$$

$$x = \frac{150}{0.03}$$

$$= 5000$$

We substitute this value of x into the formula for $p = D(x)$ to find p.

$$p = D(x) = 200 - 0.02x$$

$$p = D(5000) = 200 - 0.02(5000) = 200 - 100 = 100$$

We check this result by also substituting the same value of x into the formula for $p = S(x)$

$$p = S(x) = 0.01x + 50$$

$$p = S(5000) = 0.01(5000) + 50 = 50 + 50 = 100$$

The results agree. Conclude that the equilibrium price point is $(\bar{x}, \bar{p}) = (5000,100)$.

(b) Find the *Consumers' Surplus* at the *equilibrium price point*.

Solution:

$$CS = \int_0^{\bar{x}} [D(x) - \bar{p}]dx$$

$$= \int_0^{5000} [(200 - 0.02x) - 100]dx$$

simplify the integrand before integrating

$$= \int_0^{5000} 100 - 0.02x dx$$

$$\underset{FTC}{=} \left(\int 100 - 0.02x dx\right)\Big|_0^{5000}$$

$$= (100x - 0.01x^2 + C)|_0^{5000}$$

$$= (100(5000) - 0.01(5000)^2 + C) - (100(0) - 0.01(0)^2 + C)$$

$$= 500{,}000 - 250{,}000$$

$$= 250{,}000$$

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(c) Find the *Producers' Surplus* at the *equilibrium price point*.

Solution:

$$PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx$$

$$= \int_0^{5000} [100 - (0.01x + 50)]dx$$

simplify the integrand before integrating.

$$= \int_0^{5000} 50 - 0.01x dx$$

$$\underset{FTC}{=} \left(\int 50 - 0.01x dx\right)\Big|_0^{5000}$$

$$= (50x - 0.005x^2 + C)|_0^{5000}$$

$$= (50(5000) - 0.005(5000)^2 + C) - (50(0) - 0.005(0)^2 + C)$$

$$= 250{,}000 - 125{,}000$$

$$= 125{,}000$$

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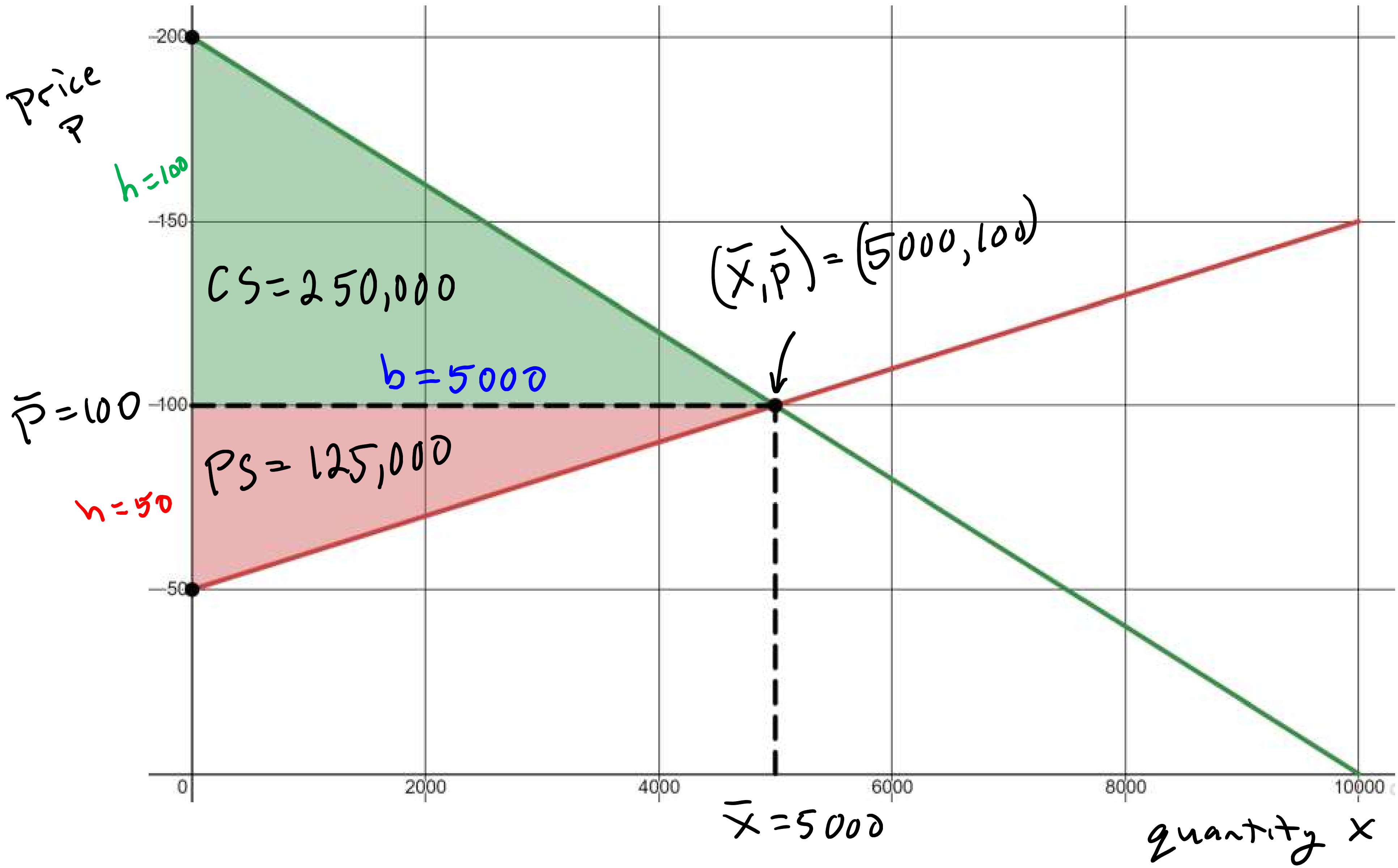
(d) Illustrate the results of **(a),(b),(c)** with a graph.

Solution:



Remark:

Observe that we can find the *Consumers' Surplus* and *Producers' Surplus* using *geometry*!

$$CS = \frac{1}{2} b \cdot h = \frac{1}{2}(5000) \cdot (100) = 5000 \cdot 50 = 250{,}000$$

$$PS = \frac{1}{2} b \cdot h = \frac{1}{2}(5000) \cdot (50) = 5000 \cdot 25 = 125{,}000$$

These numbers agree with the results that we found by doing the (harder) integral calculations.

[Example 2] (similar to Barnett 6.2#78* and Lial 7.5#35)

Suppose that

- the *Demand Price Function* is $p = D(q) = -q^2 - 6q + 1080$
- the *Supply Price Function* is $p = S(q) = q^2 + 2q + 30$

(a) Find the *equilibrium price point*.

Solution:

We set $S(q) = D(q)$ and solve for q.

$$q^2 + 2q + 30 = -q^2 - 6q + 1080$$

$$2q^2 + 8q - 1050 = 0$$

We can solve this equation using the quadratic formula using the coefficients

$$a = 2$$

$$b = 8$$

$$c = -1050$$

I will do that on the next page.

Using the quadratic formula

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(8) \pm \sqrt{(8)^2 - 4(2)(-1050)}}{2(2)}$$

$$= \frac{-8 \pm \sqrt{64 + 8400}}{4}$$

$$= \frac{-8 \pm \sqrt{8464}}{4}$$

$$= \frac{-8 \pm 92}{4}$$

We are only interested in positive q values, so our solution must be

$$q = \frac{-8 + 92}{4} = \frac{84}{4} = 21$$

We substitute this value of q into the formula for $p = D(q)$ to find p.

$$p = D(q) = -q^2 - 6q + 1080$$

$$p = D(21) = -(21)^2 - 6(21) + 1080 = -441 - 126 + 1080 = 513$$

We check this result by also substituting the same value of q into the formula for $p = S(q)$

$$p = S(q) = q^2 + 2q + 30$$

$$p = S(21) = (21)^2 + 2(21) + 30 = 441 + 42 + 30 = 513$$

The results agree. Conclude that the equilibrium price point is $(\bar{q}, \bar{p}) = (21{,}513)$

(b) Find the *Consumers' Surplus* at the *equilibrium price point*.

Solution:

$$CS = \int_0^{\bar{q}} [D(q) - \bar{p}]dq$$

$$= \int_0^{21} [(-q^2 - 6q + 1080) - 513]dq$$

simplify the integrand

$$= \int_0^{21} -q^2 - 6q + 567dq$$

$$\underset{FTC}{=} \left(\int -q^2 - 6q + 567dq\right)\Bigg|_0^{21}$$

$$= \left(-\frac{q^3}{3} - 3q^2 + 567q + C\right)\Bigg|_0^{21}$$

$$= \left(-\frac{(21)^3}{3} - 3(21)^2 + 567(21) + C\right) - \left(-\frac{(0)^3}{3} - 3(0)^2 + 567(0) + C\right)$$

$$= -\frac{9261}{3} - 1323 + 11907$$

$$= -3087 - 1323 + 11907$$

$$= 7497$$

(c) Find the *Producers' Surplus* at the *equilibrium price point*.

Solution:

$$PS = \int_0^{\bar{q}} [\bar{p} - S(x)]dq$$

$$= \int_0^{21} [513 - (q^2 + 2q + 30)]dq$$

$$= \int_0^{21} -q^2 - 2q + 483dq$$

$$\underset{FTC}{=} \left(\int -q^2 - 2q + 483dq\right)\Big|_0^{21}$$

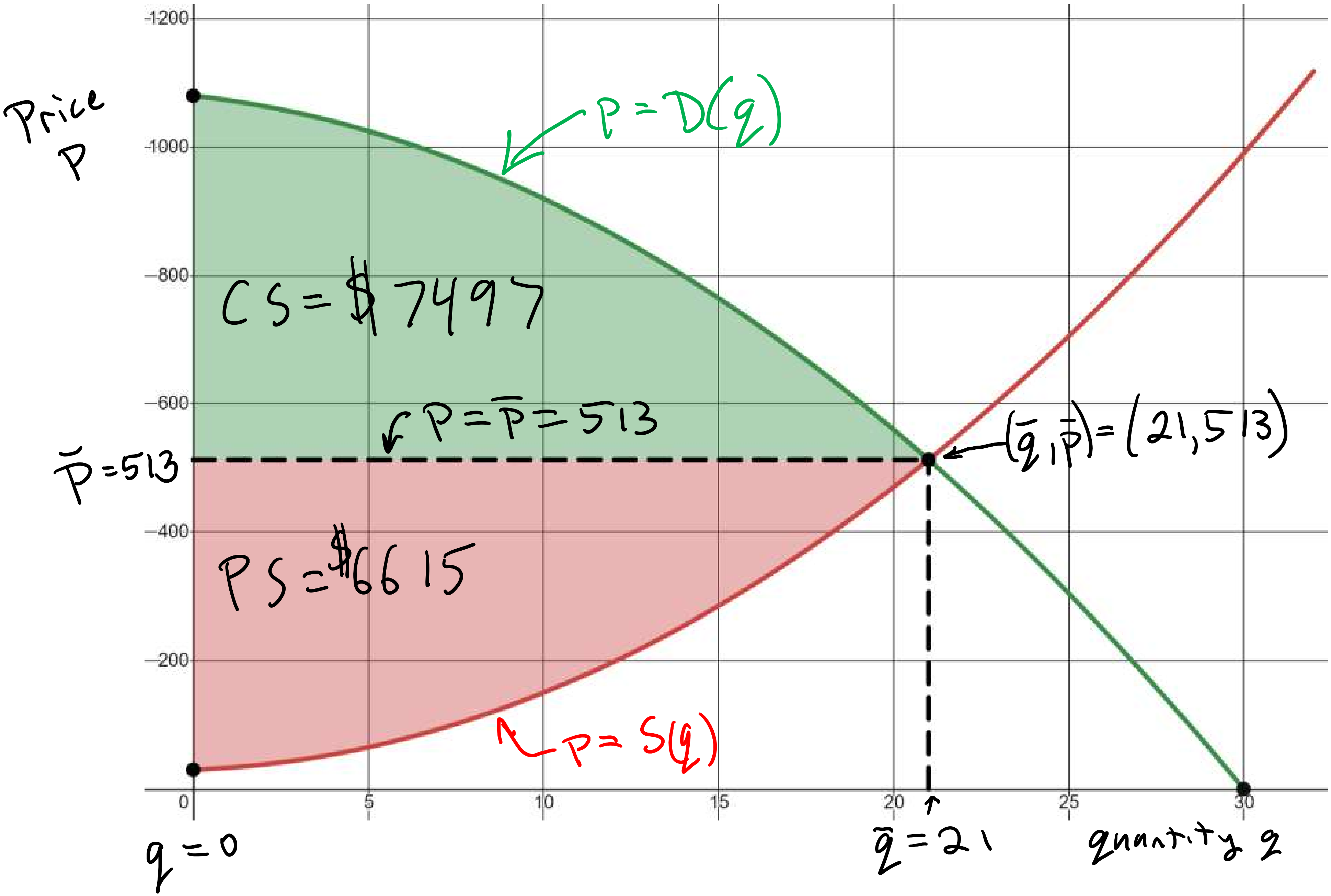
$$= \left(-\frac{q^3}{3} - q^2 + 483q + C\right)\Big|_0^{21}$$

$$= \left(-\frac{(21)^3}{3} - (21)^2 + 483(21) + C\right) - \left(-\frac{(0)^3}{3} - (0)^2 + 483(0) + C\right)$$

$$= -3087 - 441 + 10143$$

$$= 6615$$

(d) Illustrate the results of **(a),(b),(c)** with a graph.



End of [Example 2]