The Homogeneity of Covariances Assumption in MANOVA: Differential Impact of Heterogenous Variances and Covariances

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Objectives

Many applied researchers have learned that MANOVA is not robust to violations of heterogeneous covariance matrices, and that, when Box's M is statistically significant, indicating the probable violation of homogeneity, they should use Pillai's Trace (*V*) as their multivariate statistic (e.g., Meyers, Gamst, & Guarino, 2017). There have been few studies since Olson (1974, 1976) and the issue remains confusing. While this Monte Carlo experiment investigates the robustness of multivariate statistics, its focus in on the diagnosis of the assumption of homogeneity of covariance matrices using Box's M and the potential to use Levene's test to examine homogeneity of variances.

Perspectives

Olson (1976) reported that "The Pillai-Bartlett *V* test is recommended for general use. It is the most robust of the invariant tests and is sufficiently powerful…" (p. 583). However, he used some interesting examples to make his point that *V* should be preferred to Wilks' Lambda (*W*), Hotelling's Trace (*T*), or Roy's Largest Root (*R*):

> Consider an experiment with three groups of *five subjects* [emphasis added] each and a nominal significance level of .05. If there are p = 3 dependent variables and one group is sampled from a population with standard deviations three times those for the other

> groups, the actual Type I error rates for the four statistics are .09 for *V*, .13 for *W*, .15 for *T*, and .17 for *R*. (p. 583).

Stevens (1979) was particularly bothered by five subjects per group in multivariate examples. He responded with his own work that showed that *V* is only more robust in extreme conditions (e.g., 36-to-1 variance ratio, small samples). Stevens's tables did confirm that generally there is a slight robustness advantage for *V* over *T* and *W*. Stevens ultimately concluded that although "…*V* will generally be slightly more robust…" (p. 359), *W* and *T* may have a power advantage. Olson (1979) further responded and, ultimately, many textbooks over the years have simply stated that *V* should be used when Box's M test is statistically significant, implying violation of homogeneity of covariance matrices. For whatever reason, this issue has not been studied much over the decades, but there have been a few studies (e.g., Beasley & Sheehan, 1994; Finch & French, 2013).

Methods and Data Source

This study used Monte Carlo methods in R to generate and analyze data for many conditions. We generated at least 10,000 samples across three, four, and five groups. We varied variance-covariance matrices across groups as well as patterns of variances and covariances across groups. That is, we created different patterns of four types of matrices: all groups equal, groups with equal variances but different covariances, groups with equal covariances but different variances, and groups with both different variances and covariances. All group mean vectors were set equal in this robustness study of Type I error rates. All data were generated from a multivariate normal distribution. In each condition, rejections of multivariate tests were counted for calculation of Type I error or statistical power rates (e.g., rejections of the omnibus tests or the assumptions tests).

For initial simulations reported here, we ran samples sizes across groups of nearly 50. That is, for balanced group sizes we used N = 50 for all groups. For other conditions, we set roughly half of the group sizes above 50 and roughly half below (see Table 1 for an example). Also, for initial simulations, variances and covariances varied in multiple ways across groups. Table 2 provides an example of one of the more extreme covariance matrices across groups for both three and four groups. When combining number of groups, sample sizes, covariance matrices, and patterns of covariances, we simulated 10,000 samples each within over 500 conditions.

Table 1. Examples of Sample Sizes for initial simulations and some of the extreme examples of heterogeneous Variance-Covariance matrices (variances on diagonal, covariances off-diagonal) based on 4 groups

Sample Sizes

	Group1	Group2	Group3	Group4
[1,]	50	50	50	50
[2,]	54	52	50	48
[3,]	58	54	50	46
[4,]	62	56	50	44
[5,]	66	58	50	42
[6,]	70	60	50	40
[7,]	74	62	50	38
[8,]	78	64	50	36
[9,]	82	66	50	34
[10,]	86	68	50	32

Covariance Matrices

$Group1

	[,1]	[,2]	[,3]	[,4]
[1,]	1.0	0.3	0.3	0.3
[2,]	0.3	1.0	0.3	0.3
[3,]	0.3	0.3	1.0	0.3
[4,]	0.3	0.3	0.3	1.0

$Group2

	[,1]	[,2]	[,3]	[,4]
[1,]	3.0	0.5	0.5	0.5
[2,]	0.5	3.0	0.5	0.5
[3,]	0.5	0.5	3.0	0.5
[4,]	0.5	0.5	0.5	3.0

$Group3

	[,1]	[,2]	[,3]	[,4]
[1,]	5.0	0.7	0.7	0.7
[2,]	0.7	5.0	0.7	0.7
[3,]	0.7	0.7	5.0	0.7
[4,]	0.7	0.7	0.7	5.0

$Group4

	[,1]	[,2]	[,3]	[,4]
[1,]	7.0	0.9	0.9	0.9
[2,]	0.9	7.0	0.9	0.9
[3,]	0.9	0.9	7.0	0.9
[4,]	0.9	0.9	0.9	7.0

In each sample, we calculated the omnibus ANOVA using Pillai's Trace (V), Wilks' Lambda (W), Hotelling's Trace (T), and Roy's Largest Root (R). We tested homogeneity of covariance matrices using the commonly used Box's M test, but we also included Levene's test and a Bonferroni adjustment to Levene's test (alpha divided by the number of dependent variables) for homogeneity of variances. A sample was determined to have heterogeneous variances if any of the dependent variables had statistically significant Levene's tests using alpha=.05, and then similarly using the Bonferroni-adjusted alpha based on the number of variables. We used both R built-in functions and several packages (e.g., biotools, car, MASS, Matrix).

Results

We use example results from four group simulations for the tables below, but the patterns of results were essentially consistent with three and five groups and different numbers of variables. We provide illustrative results in this paper for the main conclusions.

Table 3 shows that all four tests studied (*V*, *W*, *T*, and *R*) maintained Type I error when the assumption of homogeneity of covariance matrices was true in the population. Table 3 also shows that Box's test at alpha=.05 (pBOX), at alpha=.01 (BOX_01), and at alpha=.001 (BOX001) maintained robust Type I error rates. Further, it is apparent that Levene's test is inflated if we perform the tests for all dependent variables at alpha=.05 (ANYLEV) and reject homogeneity if any significant Levene's test exists. However, using the Bonferroni correction (based on the number of dependent variables) for multiple tests in the same way maintains the family-wise Type I error rate below alpha=.05 (ANYBON). We base these robustness conclusions on the criterion which considered Type I error rates for nominal alpha=.05 robust if they fell between .04-.06, slightly less conservative than Bradley's (1979) stringent criterion of .045-.055. In all tables below, the N column corresponds to the sample sizes in Table 1.

Table 3. Type I error rates for equal covariance matrices conditions across four groups

	G	T	S	N	L	pPillai	pWilks	pHotel	pRoys	pBOX	BOX_01	BOX001	ANYLEV	ANYBON
l1	4	1	1	1	1	0.0504	0.0522	0.0531	0.3283	0.0506	0.0113	0.0015	0.1639	0.0391
l2	4	1	1	2	2	0.0505	0.0517	0.0522	0.3242	0.0518	0.0112	0.0015	0.1667	0.0410
l3	4	1	1	3	3	0.0474	0.0486	0.0494	0.3200	0.0484	0.0082	0.0003	0.1613	0.0400
l4	4	1	1	4	4	0.0522	0.0527	0.0535	0.3274	0.0476	0.0085	0.0010	0.1605	0.0376
l5	4	1	1	5	5	0.0516	0.0532	0.0543	0.3271	0.0498	0.0100	0.0009	0.1628	0.0430
l6	4	1	1	6	6	0.0479	0.0487	0.0502	0.3276	0.0497	0.0098	0.0009	0.1593	0.0437
l7	4	1	1	7	7	0.0546	0.0555	0.0569	0.3307	0.0530	0.0101	0.0006	0.1676	0.0399
l8	4	1	1	8	8	0.0494	0.0500	0.0507	0.3279	0.0479	0.0107	0.0008	0.1612	0.0406
l9	4	1	1	9	9	0.0518	0.0528	0.0540	0.3257	0.0478	0.0090	0.0008	0.1598	0.0417
l10	4	1	1	10	10	0.0508	0.0519	0.0530	0.3239	0.0486	0.0089	0.0007	0.1598	0.0403

Table 4 shows that as covariance matrices deviated across groups (both variances and covariances unequal) and sample sizes become more unbalanced, the Type I error rates tended to increase. In these conditions, variances and covariances were inversely related to sample sizes (i.e., larger variances and covariances occurring with smaller sample sizes). Table 4 also shows that all alpha-level versions of Box's M and both alpha-level versions of Levene's test are very powerful when both the variances and covariances differ across groups.

Table 4. Type I error rates for unequal covariance matrices conditions (both variance and covariances unequal) across four groups

	G	T	S	N	L	pPillai	pWilks	pHotel	pRoys	pBOX	BOX_01	BOX001	ANYLEV	ANYBON
l151	4	8	4	1	151	0.0635	0.0667	0.0689	0.3677	1	1	1	1	1
l152	4	8	4	2	152	0.0716	0.0743	0.0772	0.3957	1	1	1	1	1
l153	4	8	4	3	153	0.0865	0.0887	0.0920	0.4186	1	1	1	1	1
l154	4	8	4	4	154	0.0987	0.1016	0.1054	0.4466	1	1	1	1	1
l155	4	8	4	5	155	0.1140	0.1171	0.1209	0.4763	1	1	1	1	1
l156	4	8	4	6	156	0.1317	0.1368	0.1409	0.4931	1	1	1	1	1
l157	4	8	4	7	157	0.1472	0.1498	0.1545	0.5136	1	1	1	1	1
l158	4	8	4	8	158	0.1693	0.1730	0.1768	0.5442	1	1	1	1	1
l159	4	8	4	9	159	0.1822	0.1851	0.1888	0.5626	1	1	1	1	1
l160	4	8	4	10	160	0.2004	0.2040	0.2072	0.5937	1	1	1	1	1

Table 5 shows that when covariances are equal across groups but variances differ, the Type I error rates become. All versions of Box's M and both versions of Levene's test are very powerful when only variances differ across groups. The Type I error rate inflation of the multivariate statistics was essentially the same as that found in Table 4. Table 5 shows that different patterns of covariances across the groups do not seem to impact the level of inflation of Type I error rates dramatically.

Table 5. Type I error rates for equal covariances but unequal variances across four groups

Variances increase as group sample sizes decrease

	G	T	S	N	L	pPillai	pWilks	pHotel	pRoys	pBOX	BOX_01	BOX001	ANYLEV	ANYBON
1141	4	8	3	1	141	0.0639	0.0661	0.0681	0.3603	1	1	1	1	1
1142	4	8	3	2	142	0.0758	0.0791	0.0818	0.3942	1	1	1	1	1
1143	4	8	3	3	143	0.0864	0.0905	0.0940	0.4267	1	1	1	1	1
1144	4	8	3	4	144	0.0997	0.1035	0.1071	0.4573	1	1	1	1	1
1145	4	8	3	5	145	0.1192	0.1232	0.1273	0.4733	1	1	1	1	1
1146	4	8	3	6	146	0.1342	0.1371	0.1414	0.4894	1	1	1	1	1
1147	4	8	3	7	147	0.1516	0.1547	0.1587	0.5233	1	1	1	1	1
1148	4	8	3	8	148	0.1619	0.1666	0.1713	0.5426	1	1	1	1	1
1149	4	8	3	9	149	0.1901	0.1944	0.1981	0.5679	1	1	1	1	1
1150	4	8	3	10	150	0.2000	0.2036	0.2073	0.5907	1	1	1	1	1

Variance for Group 1 = 2, Variances for Group 3 = 4, and Variances for 3 & 4 > 1 & 2

	G	T	S	N	L	pPillai	pWilks	pHotel	pRoys	pBOX	BOX_01	BOX001	ANYLEV	ANYBON
1101	4	6	3	1	101	0.0633	0.0659	0.0692	0.3701	1	1	1	1	1
1102	4	6	3	2	102	0.0690	0.0712	0.0744	0.3860	1	1	1	1	1
1103	4	6	3	3	103	0.0767	0.0793	0.0823	0.3996	1	1	1	1	1
1104	4	6	3	4	104	0.0944	0.0970	0.0991	0.4302	1	1	1	1	1
1105	4	6	3	5	105	0.0999	0.1024	0.1057	0.4489	1	1	1	1	1
1106	4	6	3	6	106	0.1201	0.1229	0.1263	0.4689	1	1	1	1	1
1107	4	6	3	7	107	0.1318	0.1351	0.1382	0.4936	1	1	1	1	1
1108	4	6	3	8	108	0.1411	0.1447	0.1482	0.5102	1	1	1	1	1
1109	4	6	3	9	109	0.1523	0.1560	0.1598	0.5285	1	1	1	1	1
1110	4	6	3	10	110	0.1735	0.1770	0.1797	0.5477	1	1	1	1	1

Finally, Table 6 shows condition with equal variances in all groups and illustrates that almost none of the inflated Type I error rates across both unequal variances and covariances (like Table 4) resulted from only heterogeneous covariances when the groups had equal variances. Table 6 shows that the Bonferroni-adjusted version of Levene's test maintains robustness when variances are equal even in the presence of unequal covariances, but the unadjusted version of Levene's test again becomes inflated. Table 6 also shows that Box's M has different power levels depending on the character of the covariances. Curiously, there is slight indication that as sample sizes become more diverse the Type I error rates became more conservative.

Table 6. Type I error rates for equal variances but unequal covariances across four groups

Variances increase as group sample sizes decrease

	G	T	S	N	L	pPillai	pWilks	pHotel	pRoys	pBOX	BOX_01	BOX001	ANYLEV	ANYBON
1131	4	8	2	1	131	0.0624	0.0642	0.0666	0.3459	1	1	1	0.1414	0.0355
1132	4	8	2	2	132	0.0532	0.0544	0.0557	0.3265	1	1	1	0.1446	0.0345
1133	4	8	2	3	133	0.0462	0.0486	0.0500	0.3137	1	1	1	0.1435	0.0378
1134	4	8	2	4	134	0.0437	0.0445	0.0462	0.3040	1	1	1	0.1433	0.0370
1135	4	8	2	5	135	0.0390	0.0404	0.0420	0.2846	1	1	1	0.1453	0.0389
1136	4	8	2	6	136	0.0347	0.0354	0.0360	0.2851	1	1	1	0.1463	0.0375
1137	4	8	2	7	137	0.0339	0.0354	0.0367	0.2611	1	1	1	0.1459	0.0356
1138	4	8	2	8	138	0.0321	0.0331	0.0344	0.2627	1	1	1	0.1426	0.0329
1139	4	8	2	9	139	0.0287	0.0294	0.0308	0.2567	1	1	1	0.1473	0.0364
1140	4	8	2	10	140	0.0314	0.0320	0.0326	0.2481	1	1	1	0.1465	0.0371

Variance for Group 1 = 2, Variances for Group 3 = 4, and Variances for 3 & 4 > 1 & 2

	G	T	S	N	L	pPillai	pWilks	pHotel	pRoys	pBOX	BOX_01	BOX001	ANYLEV	ANYBON
191	4	6	2	1	91	0.0495	0.0507	0.0525	0.3316	0.3212	0.1311	0.0351	0.1577	0.0372
192	4	6	2	2	92	0.0476	0.0488	0.0500	0.3218	0.3138	0.1329	0.0326	0.1612	0.0394
193	4	6	2	3	93	0.0427	0.0440	0.0452	0.3229	0.3232	0.1350	0.0317	0.1585	0.0372
194	4	6	2	4	94	0.0442	0.0456	0.0464	0.3196	0.3227	0.1297	0.0307	0.1584	0.0378
195	4	6	2	5	95	0.0434	0.0452	0.0463	0.3130	0.3299	0.1393	0.0354	0.1673	0.0433
196	4	6	2	6	96	0.0434	0.0446	0.0452	0.3025	0.3299	0.1436	0.0372	0.1653	0.0389
197	4	6	2	7	97	0.0448	0.0459	0.0469	0.3081	0.3348	0.1408	0.0345	0.1700	0.0435
198	4	6	2	8	98	0.0428	0.0435	0.0444	0.3035	0.3340	0.1368	0.0369	0.1688	0.0413
199	4	6	2	9	99	0.0392	0.0406	0.0419	0.3050	0.3331	0.1396	0.0312	0.1632	0.0410
1100	4	6	2	10	100	0.0419	0.0430	0.0438	0.3010	0.3328	0.1411	0.0363	0.1597	0.0378

Conclusions

Ultimately, we determined that there was not much difference in Type I error rates between *V*, *W*, *T*, and *R*. We were able to see that using Box's M as a preliminary test does not serve much useful purpose. For example, it was typically powerful in unequal covariance conditions, which does not impact Type I error rates of the multivariate statistics. Although not presented in the tables above, the Type I error rate for Pillai's trace conditionally after running Box's M was not usually better than simply running Pillai's Trace unconditionally.

The argument to use Pillai when Box's M is statistically significant does not appear justified in our results. Pillai's Trace is not much more robust than Wilks' Lambda and Hotelling's Trace—but is very slightly more robust, particularly with small sample sizes. We found that even just a few cases different per group caused Type I error to become inflated in some conditions. Indeed, even with equal sample sizes the Type I error was outside our robustness criteria for some conditions.

However, what is not reported loudly in the literature is that when the variances differ across groups along with sample sizes differing across groups (the inverse relationship between variances and sample sizes), NONE of the multivariate statistics is robust. Further, we found, like Beasley and Sheehan (1994), that it is the heterogeneous variances that have the larger impact on Type I error inflation rather than heterogeneous covariances.

Most importantly, we found that using a Bonferroni adjustment to Levene's test can be used to identify heterogenous variances across groups in the multivariate situation like in the univariate situation. Using the Bonferroni-adjusted Levene's in this way may run counter to the multivariate relationships among dependent variables, but our results suggest that the approach will control Type I error. More work will need to be done to study power. Our recommendation is not to worry about Box's M but rather to test equality of variances using a Bonferroni-corrected Levene's test. If any of the variables has statistically significantly unequal variances, then concern and limitations should be raised regarding the robustness of any of the multivariate statistics.

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Appendices

	Vars	Groups	T	S	N	L	pPillai	pWilks	pHotel	pRoys	BOX_05	BOX_01	BOX001
l10	4	4	1	1	10	10	0.051	0.052	0.053	0.324	0.049	0.009	0.001
l20	4	4	1	2	10	20	0.048	0.048	0.050	0.326	0.051	0.012	0.002
l30	4	4	1	3	10	30	0.048	0.049	0.050	0.324	0.057	0.010	0.001
l40	4	4	1	4	10	40	0.050	0.050	0.052	0.322	0.048	0.008	0.001
l50	4	4	2	1	10	50	0.050	0.051	0.052	0.321	0.051	0.011	0.001
l90	4	4	3	1	10	90	0.050	0.050	0.051	0.328	0.052	0.010	0.001
l130	4	4	4	1	10	130	0.046	0.047	0.048	0.326	0.048	0.010	0.001
l170	4	4	5	1	10	170	0.046	0.048	0.048	0.318	0.051	0.011	0.001
l210	4	4	6	1	10	210	0.049	0.050	0.051	0.329	0.048	0.009	0.001
l250	4	4	7	1	10	250	0.048	0.050	0.051	0.328	0.048	0.008	0.001
l290	4	4	8	1	10	290	0.050	0.052	0.052	0.319	0.048	0.009	0.001
l330	4	4	9	1	10	330	0.055	0.056	0.058	0.326	0.046	0.011	0.001
l370	4	4	10	1	10	370	0.047	0.048	0.048	0.314	0.052	0.011	0.001
l410	4	4	11	1	10	410	0.048	0.048	0.050	0.332	0.052	0.011	0.001

	ANYBON	ANYLEV	BOX_05P	BOX_01P	BOX001P	ANYBONP	ANYLEVP
l10	0.042	0.160	0.050	0.050	0.051	0.050	0.050
l20	0.043	0.169	0.048	0.048	0.048	0.049	0.049
l30	0.040	0.164	0.048	0.048	0.048	0.048	0.048
l40	0.041	0.165	0.049	0.049	0.049	0.050	0.050
l50	0.037	0.158	0.050	0.050	0.050	0.049	0.049
l90	0.040	0.164	0.050	0.050	0.050	0.049	0.050
l130	0.039	0.166	0.046	0.046	0.046	0.047	0.048
l170	0.043	0.166	0.047	0.046	0.046	0.047	0.046
l210	0.040	0.161	0.049	0.049	0.049	0.048	0.047
l250	0.042	0.166	0.048	0.048	0.048	0.048	0.048
l290	0.041	0.163	0.050	0.050	0.050	0.050	0.051
l330	0.041	0.164	0.054	0.054	0.055	0.055	0.054
l370	0.041	0.168	0.047	0.047	0.047	0.047	0.046
l410	0.041	0.165	0.048	0.048	0.048	0.048	0.048

	Vars	Groups	T	S	N	L	pPillai	pWilks	pHotel	pRoys	BOX_05	BOX_01	BOX001
l60	4	4	2	2	10	60	0.056	0.057	0.058	0.341	0.292	0.117	0.027
l100	4	4	3	2	10	100	0.050	0.051	0.051	0.323	0.232	0.081	0.016
l140	4	4	4	2	10	140	0.044	0.046	0.047	0.309	0.158	0.051	0.009
l180	4	4	5	2	10	180	0.041	0.042	0.043	0.303	0.354	0.154	0.043
l220	4	4	6	2	10	220	0.043	0.043	0.044	0.298	0.328	0.130	0.031
l260	4	4	7	2	10	260	0.040	0.041	0.042	0.290	0.461	0.224	0.066
l300	4	4	8	2	10	300	0.026	0.026	0.027	0.241	1.000	1.000	1.000
l340	4	4	9	2	10	340	0.056	0.057	0.058	0.350	0.333	0.146	0.036
l380	4	4	10	2	10	380	0.083	0.086	0.087	0.402	0.993	0.964	0.869
l420	4	4	11	2	10	420	0.145	0.148	0.150	0.490	1.000	1.000	1.000

	ANYBON	ANYLEV	BOX_05P	BOX_01P	BOX001P	ANYBONP	ANYLEVP
l60	0.045	0.165	0.058	0.056	0.056	0.057	0.057
l100	0.041	0.160	0.049	0.050	0.051	0.049	0.048
l140	0.042	0.163	0.044	0.044	0.044	0.045	0.044
l180	0.038	0.157	0.040	0.041	0.041	0.040	0.040
l220	0.036	0.153	0.040	0.041	0.043	0.043	0.043
l260	0.043	0.162	0.044	0.040	0.039	0.041	0.042
l300	0.038	0.143	NaN	NaN	NaN	0.025	0.026
l340	0.037	0.162	0.055	0.057	0.055	0.056	0.056
l380	0.041	0.161	0.119	0.095	0.090	0.083	0.082
l420	0.037	0.151	NaN	NaN	NaN	0.146	0.147

Appendices (cont.)

	Vars	Groups	T	S	N	L	pPillai	pWilks	pHotel	pRoys	BOX_05	BOX_01	BOX001
l70	4	4	2	3	10	70	0.046	0.047	0.049	0.327	1	1	1.000
l110	4	4	3	3	10	110	0.099	0.103	0.106	0.427	1	1	1.000
l150	4	4	4	3	10	150	0.180	0.184	0.188	0.547	1	1	0.999
l190	4	4	5	3	10	190	0.103	0.105	0.107	0.448	1	1	1.000
l230	4	4	6	3	10	230	0.174	0.178	0.182	0.554	1	1	1.000
l270	4	4	7	3	10	270	0.162	0.164	0.166	0.538	1	1	1.000
l310	4	4	8	3	10	310	0.198	0.203	0.206	0.585	1	1	1.000
l350	4	4	9	3	10	350	0.020	0.021	0.022	0.222	1	1	1.000
l390	4	4	10	3	10	390	0.014	0.016	0.017	0.211	1	1	1.000
l430	4	4	11	3	10	430	0.016	0.017	0.018	0.218	1	1	1.000

	ANYBON	ANYLEV	BOX_05P	BOX_01P	BOX001P	ANYBONP	ANYLEVP
l70	1.000	1	NaN	NaN	NaN	NaN	NaN
l110	1.000	1	NaN	NaN	NaN	NaN	NaN
l150	0.999	1	NaN	1	0.286	0.3	NaN
l190	1.000	1	NaN	NaN	NaN	NaN	NaN
l230	1.000	1	NaN	NaN	NaN	NaN	NaN
l270	1.000	1	NaN	NaN	NaN	NaN	NaN
l310	1.000	1	NaN	NaN	NaN	NaN	NaN
l350	1.000	1	NaN	NaN	NaN	NaN	NaN
l390	1.000	1	NaN	NaN	NaN	NaN	NaN
l430	1.000	1	NaN	NaN	NaN	NaN	NaN

	Vars	Groups	T	S	N	L	pPillai	pWilks	pHotel	pRoys	BOX_05	BOX_01	BOX001
l70	4	4	2	3	10	70	0.046	0.047	0.049	0.327	1	1	1.000
l110	4	4	3	3	10	110	0.099	0.103	0.106	0.427	1	1	1.000
l150	4	4	4	3	10	150	0.180	0.184	0.188	0.547	1	1	0.999
l190	4	4	5	3	10	190	0.103	0.105	0.107	0.448	1	1	1.000
l230	4	4	6	3	10	230	0.174	0.178	0.182	0.554	1	1	1.000
l270	4	4	7	3	10	270	0.162	0.164	0.166	0.538	1	1	1.000
l310	4	4	8	3	10	310	0.198	0.203	0.206	0.585	1	1	1.000
l350	4	4	9	3	10	350	0.020	0.021	0.022	0.222	1	1	1.000
l390	4	4	10	3	10	390	0.014	0.016	0.017	0.211	1	1	1.000
l430	4	4	11	3	10	430	0.016	0.017	0.018	0.218	1	1	1.000

	ANYBON	ANYLEV	BOX_05P	BOX_01P	BOX001P	ANYBONP	ANYLEVP
l70	1.000	1	NaN	NaN	NaN	NaN	NaN
l110	1.000	1	NaN	NaN	NaN	NaN	NaN
l150	0.999	1	NaN	1	0.286	0.3	NaN
l190	1.000	1	NaN	NaN	NaN	NaN	NaN
l230	1.000	1	NaN	NaN	NaN	NaN	NaN
l270	1.000	1	NaN	NaN	NaN	NaN	NaN
l310	1.000	1	NaN	NaN	NaN	NaN	NaN
l350	1.000	1	NaN	NaN	NaN	NaN	NaN
l390	1.000	1	NaN	NaN	NaN	NaN	NaN
l430	1.000	1	NaN	NaN	NaN	NaN	NaN