

# Separable Representation of Proton-Nucleus Optical Potentials

L. Hlophe<sup>(a),\*</sup>, V. Eremenko<sup>(a,e)</sup>, Ch. Elster<sup>(a),†</sup> and F.M. Nunes<sup>(b)</sup>, G. Arbanas<sup>(c)</sup>, J.E. Escher<sup>(d)</sup>, I.J. Thompson<sup>(d)</sup>

*(a) Institute of Nuclear and Particle Physics, and Department of Physics and Astronomy, Ohio University, Athens, OH 45701*

*(b) National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy,  
Michigan State University, East Lansing, MI 48824, USA*

*(c) Reactor and Nuclear Systems Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA*

*(d) Lawrence Livermore National Laboratory L-414, Livermore,  
CA 94551, USA (e) D.V. Skobeltsyn Institute of Nuclear Physics,  
M.V. Lomonosov Moscow State University, Moscow, 119991, Russia*

(The TORUS Collaboration)

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Recently, a new approach for solving the three-body problem for (d,p) reactions in the Coulomb-distorted basis in momentum space was proposed. Important input quantities for such calculations are the scattering matrix elements for proton- and neutron-nucleus scattering. We present a generalization of the Ernst-Shakin-Thaler scheme in which a momentum space separable representation of proton-nucleus scattering matrix elements can be calculated in the Coulomb basis. The viability of this method is demonstrated by comparing S-matrix elements obtained for  $p+^{48}\text{Ca}$  and  $p+^{208}\text{Pb}$  for a phenomenological optical potential with corresponding coordinate space calculations.

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## I. INTRODUCTION

Deuteron induced nuclear reactions are attractive from an experimental as well as theoretical point of view for probing the structure of exotic nuclei and as an indirect tool in astrophysics (see e.g. [1]). From a theoretical perspective, (d,p) reactions are attractive since the scattering problem can be viewed as an effective three-body problem [2]. One of the most challenging aspects of solving the three-body problem for nuclear reactions is the repulsive Coulomb interaction between the nucleus and the proton. While exact calculations of (d,p) reactions based on Faddeev equations in the Alt-Grassberger-Sandhas (AGS) [3] formulation can be carried out [4] for very light nuclei, this is not the case for heavier nuclei with higher charges. The reason for this shortcoming is rooted in implementations of the Faddeev-AGS equations that rely on a screening and renormalization procedure [5, 6], which leads to increasing technical difficulties in computing (d,p) reactions with heavier nuclei [7].

In Ref. [8], a three-body theory for (d,p) reactions is derived, where no screening of the Coulomb force is introduced. Therein, the Faddeev-AGS equations are cast in the Coulomb-distorted partial-wave representation, instead of the plane-wave basis. The interactions in the different two-body subsystems, including the neutron- and proton-nucleus interactions, are assumed to be of separable form.

Separable forms for nucleon-nucleus interactions have been considered in the past (e.g. [9, 10]), but are usually of a rank-1 Yamaguchi form and are intended to represent the nuclear forces up to a few MeV. This is not

sufficient for scattering of heavy nuclei up to tens of MeV. In addition, adjusting the parameters of Yamaguchi-type neutron-nucleus form factors to obtain proton-nucleus form factors is not very practical when considering a larger variety of nuclei. Therefore, a systematic scheme for deriving separable representations for proton-nucleus optical potentials is needed.

In Ref. [11] we derived a separable representation of phenomenological neutron-nucleus optical potentials, based on a generalization of the Ernst, Shakin and Thaler (EST) scheme for non-hermitian interactions. In Ref. [12] we presented the first test calculations of form factors in the momentum-space Coulomb basis, using the neutron-nucleus interaction developed in [11]. In this work we generalize these studies for proton-nucleus interactions.

The derivations in the original EST work laid out in [13] set up the scattering problem in a complete plane-wave basis, whereas in this work we need to use a complete Coulomb basis. Consequently, when working in momentum space, we require a solution of the momentum space scattering equation in the Coulomb basis exists. We solve the momentum space Lippmann-Schwinger (LS) equation in the Coulomb basis, following the method introduced in Ref. [14] and successfully applied in proton-nucleus scattering calculations with microscopic optical potentials in Ref. [15]. We note that a separable expansion for local potentials with Coulomb interactions was first derived by Adhikari [16] and applied to proton-proton scattering. However, it has never been applied to proton scattering from heavier nuclei.

In Sec. II we sketch the important steps needed to derive a separable representation of a phenomenological global optical potential in the momentum-space Coulomb basis for proton-nucleus scattering. Our numerical calculations of S-matrix elements for proton scattering from  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$  at selected laboratory kinetic energies are

\* lh421709@ohio.edu

† elster@ohio.edu

discussed in Sec. III, along with the behavior of the form factors as a function of the external momentum. Finally, we summarize our work in Sec. IV.

## II. FORMAL CONSIDERATIONS

The scattering between a proton and a nucleus is governed by a potential

$$w = v^C + u^s, \quad (1)$$

where  $v^C$  is the repulsive Coulomb potential and  $u^s$  an arbitrary short range potential. For the proton-nucleus system  $u^s$  consists of an optical potential, which describes the nuclear interactions, and a short-ranged Coulomb potential, traditionally parameterized as the potential of a charged sphere with radius  $R_0$  of which the point Coulomb force is subtracted [17]. Since the scattering problem governed by the point Coulomb force has an analytic solution, the scattering amplitude for elastic scattering between a proton and a spin-zero nucleus is obtained as the sum of the Rutherford amplitude  $f^C(E_{p_0}, \theta)$  and the Coulomb distorted nuclear amplitude given by

$$M^{CN}(E_{p_0}, \theta) = f^{CN}(E_{p_0}, \theta) + \hat{\sigma} \cdot \hat{\mathbf{n}} g^{CN}(E_{p_0}, \theta), \quad (2)$$

with

$$f^{CN}(E_{p_0}, \theta) = -\pi\mu \sum_{l=0}^{\infty} e^{2i\sigma_l(E_{p_0})} P_l(\cos\theta) \times \left[ (l+1) \langle p_0 | \tau_{l+}^{CN}(E_{p_0}) | p_0 \rangle + l \langle p_0 | \tau_{l-}^{CN}(E_{p_0}) | p_0 \rangle \right], \quad (3)$$

$$g^{CN}(E_{p_0}, \theta) = -\pi\mu \sum_{l=0}^{\infty} e^{2i\sigma_l(E_{p_0})} P_l^1(\cos\theta) \times \left[ \langle p_0 | \tau_{l+}^{CN}(E_{p_0}) | p_0 \rangle - \langle p_0 | \tau_{l-}^{CN}(E_{p_0}) | p_0 \rangle \right]. \quad (4)$$

Here  $E_{p_0} = p_0^2/2\mu$  is the center-of-mass (c.m.) scattering energy which defines the on-shell momentum  $p_0$ , and  $\sigma_l = \arg\Gamma(1+l+i\eta)$  is the Coulomb phase shift. The Sommerfeld parameter is given by  $\eta = \alpha Z_1 Z_2 \mu / p_0$  with  $Z_1$  and  $Z_2$  being the atomic numbers of the particles, and  $\alpha$  the Coulomb coupling constant. The unit vector  $\hat{\mathbf{n}}$  is normal to the scattering plane, and  $\hat{\sigma}/2$  is the spin operator. The subscripts  $'+$ ' and  $'-'$  correspond to a total angular momentum  $j = l + 1/2$  and  $j = l - 1/2$ . All calculations shown in this work refer to  $j = l + 1/2$ . Suppressing the total angular momentum indices for simplicity, the Coulomb distorted nuclear  $t$ -matrix element is given by  $\langle p_0 | \tau_l^{CN}(E_{p_0}) | p_0 \rangle$ , which is the solution of a LS type equation,

$$\langle p | \tau_l^{CN}(E_{p_0}) | p_0 \rangle = \langle p | u_l^s | p_0 \rangle + \int p'^2 dp' \langle p | u_l^s | p' \rangle \langle p' | g_c(E_{p_0} + i\varepsilon) | p' \rangle \langle p' | \tau_l^{CN}(E_{p_0}) | p_0 \rangle. \quad (5)$$

Here  $g_c^{-1}(E_{p_0} + i\varepsilon) = E_{p_0} + i\varepsilon - H_0 - v^C$  is the Coulomb Green's function, and  $H_0$  the free Hamiltonian. The Coulomb distorted nuclear  $t$ -matrix element

$\langle p | \tau_l^{CN}(E_{p_0}) | p_0 \rangle$  is related to the proton-nucleus  $t$ -matrix  $\langle p | t_l(E_{p_0}) | p_0 \rangle$  by the familiar two-potential formula

$$\langle p | t_l(E_{p_0}) | p_0 \rangle = \langle p | t_l^C(E_{p_0}) | p_0 \rangle + e^{2i\sigma_l(E_{p_0})} \langle p | \tau_l^{CN}(E_{p_0}) | p_0 \rangle, \quad (6)$$

where  $\langle p | t_l^C(E_{p_0}) | p_0 \rangle$  is the point Coulomb  $t$ -matrix.

When the integral equation in Eq. (6) is solved in the basis of Coulomb eigenfunctions,  $g_c$  acquires the form of a free Green's function and the difficulty of solving it is shifted to evaluating the potential matrix elements in this basis.

For deriving a separable representation of the Coulomb distorted proton-nucleus  $t$ -matrix element, we generalize the approach suggested by Ernst, Shakin, and Thaler (EST) [13], to the charged particle case. The basic idea behind the EST construction of a separable representation of a given potential is that the wave functions calculated with this potential and the corresponding separable potential agree at given fixed scattering energies  $E_i$ , the EST support points. The formal derivations of [13] use the plane wave basis, which is standard for scattering involving short-range potentials. However, the EST scheme does not depend on the basis and can equally well be carried out in the basis of Coulomb scattering wave functions. In order to generalize the EST approach to charged-particle scattering, one needs to be able to obtain the scattering wave functions or half-shell  $t$ -matrices from a given potential in the Coulomb basis, and then construct the corresponding separable representation thereof.

### A. The half-shell $t$ -matrices in the Coulomb basis

In order to calculate the half-shell  $t$ -matrix of Eq. (6), we evaluate the integral equation in the Coulomb basis as suggested in [14], and note that in this case the Coulomb Green's function behaves like a free Green's function. Taking  $|\Phi_{l,p}^c\rangle$  to represent the partial wave Coulomb eigenstate, the LS equation becomes

$$\begin{aligned} \langle \Phi_{l,p}^c | \tau_l^{CN}(E_{p_0}) | \Phi_{l,p_0}^c \rangle &= \langle \Phi_{l,p}^c | u_l^s | \Phi_{l,p_0}^c \rangle + \\ &\int_0^\infty \langle \Phi_{l,p}^c | u_l^s | \Phi_{l,p'}^c \rangle \frac{p'^2 dp'}{E_{p_0} - E_{p'} + i\varepsilon} \langle \Phi_{l,p'}^c | \tau_l^{CN}(E_{p_0}) | \Phi_{l,p_0}^c \rangle \\ &\equiv \langle p | \tau_l^{CN}(E_{p_0}) | p_0 \rangle, \end{aligned} \quad (7)$$

which defines the Coulomb distorted nuclear  $t$ -matrix of Eq. (6). To determine the short-range potential matrix element, we follow Ref. [14] and insert a complete set of position space eigenfunctions

$$\begin{aligned} \langle \Phi_{l,p'}^c | u_l^s | \Phi_{l,p}^c \rangle &= \frac{2}{\pi} \int_0^\infty \langle \Phi_{l,p'}^c | r' \rangle r'^2 dr' \langle r' | u_l^s | r \rangle r^2 dr \langle r | \Phi_{l,p}^c \rangle \\ &= \frac{2}{\pi p' p} \int_0^\infty r r' dr dr' F_l(\eta', p' r') \langle r' | u_l^s | r \rangle F_l(\eta, p r). \end{aligned} \quad (8)$$

The partial wave Coulomb functions are given in coordinate space as

$$\langle r | \Phi_{l,p}^c \rangle \equiv \frac{e^{i\sigma_l(p)} F_l(\eta, pr)}{pr}, \quad (9)$$

where  $F_l(\eta, pr)$  are the standard Coulomb functions [18] and  $\eta(\eta')$  is the Sommerfeld parameter determined with momentum  $p(p')$ .

For our application we consider phenomenological optical potentials of Woods-Saxon form which are local in coordinate space. Thus the momentum space potential matrix elements simplify to

$$\langle \Phi_{l,p'}^c | u_l^s | \Phi_{l,p}^c \rangle = \frac{2}{\pi p' p} \int_0^\infty dr F_l(\eta', p'r) u_l^s(r) F_l(\eta, pr) \quad (10)$$

We compute these matrix elements for the short-range piece of the CH89 phenomenological global optical potential [17], which consists of the nuclear and short range Coulomb potential. The nuclear potential is parameterized using Woods-Saxon functions. For the short range Coulomb interaction, the potential of a uniformly charged sphere is assumed, from which the point Coulomb force is subtracted. The integral can be carried out with standard methods, since  $u_l^s(r)$  is short ranged and the coordinate space Coulomb wavefunctions are well defined. The accuracy of this integral can be tested by replacing the Coulomb functions with spherical Bessel functions and comparing the resulting matrix elements to the partial-wave decomposition of the semi-analytic Fourier transform used in [11]. For the cases under study, and a maximum radius of 14 fm, 300 grid points are sufficient to obtain matrix elements with a precision of six significant digits.

### B. EST representation of the proton-nucleus $t$ -matrix in the Coulomb basis

Extending the EST separable representation to the Coulomb basis involves replacing the neutron-nucleus half-shell  $t$ -matrix in Eqs. (14)-(16) of Ref. [11] by the Coulomb distorted nuclear half-shell  $t$ -matrix. This leads to the separable Coulomb distorted nuclear  $t$ -matrix

$$\tau_l^{CN}(E_{p_0}) = \sum_{i,j} u_l^s | f_{l,k_{E_i}}^c \rangle \tau_{ij}^c(E_{p_0}) \langle f_{l,k_{E_j}}^{c*} | u_l^s, \quad (11)$$

with  $\tau_{ij}^c(E_{p_0})$  being constrained by

$$\sum_i \langle f_{l,k_{E_n}}^{c*} | u_l^s - u_l^s g_c(E_{p_0}) u_l^s | f_{l,k_{E_i}}^c \rangle \tau_{ij}^c(E) = \delta_{ni} \quad (12)$$

$$\sum_j \tau_{ij}^{CN}(E_{p_0}) \langle f_{l,k_{E_j}}^{c*} | u_l^s - u_l^s g_c(E_{p_0}) u_l^s | f_{l,k_{E_k}}^c \rangle = \delta_{ik}.$$

Here  $| f_{l,k_{E_i}}^c \rangle$  and  $| f_{l,k_{E_i}}^{c*} \rangle$  are the regular radial scattering wave functions corresponding to the short range potentials  $u_l^s$  and  $(u_l^s)^*$  at energy  $E_i$ . The constraints of

$l$	separable		p-space		r-space	
0	-0.0512	0.3765	-0.0518	0.3768	-0.0523	0.3767
2	0.3805	0.0420	0.3809	0.0421	0.3808	0.0427
6	-0.0445	0.0170	-0.0457	0.0118	-0.0462	0.0111
10	0.9818	0.0248	0.9814	0.0253	0.9814	0.0253

TABLE I. The partial wave  $S$ -matrix elements obtained from the CH89 [17] phenomenological optical potential for  $j = l + 1/2$  for selected angular momenta  $l$  calculated for  $p+^{48}\text{Ca}$  elastic scattering at  $E_{lab} = 38$  MeV.

Eqs. (13) ensure that, at the EST support points, the exact and separable Coulomb distorted nuclear half-shell  $t$ -matrices are identical. We want to point out that the generalization of the EST scheme to complex potentials [11] is not affected by changing the basis from plane waves to Coulomb scattering states. The separable Coulomb distorted nuclear  $t$ -matrix elements are given by

$$\begin{aligned} \langle p' | \tau_l^{CN}(E_{p_0}) | p \rangle &\equiv \sum_{i,j} h_{l,i}^c(p') \tau_{ij}^c(E_{p_0}) h_{l,j}^c(p) = \\ &= \sum_{i,j} \langle \Phi_{l,p'}^c | u_l^s | f_{l,k_{E_i}}^c \rangle \tau_{ij}^c(E_{p_0}) \langle f_{l,k_{E_j}}^{c*} | u_l^s | \Phi_{l,p}^c \rangle, \quad (13) \end{aligned}$$

where the form factor

$$\begin{aligned} h_{l,i}^c(p) &\equiv \langle \Phi_{l,p}^c | u_l^s | f_{l,k_{E_i}}^c \rangle \\ &= \langle f_{l,k_{E_i}}^{c*} | u_l^s | \Phi_{l,p}^c \rangle = \langle p | \tau_l^{CN}(E_i) | k_{E_i} \rangle \end{aligned} \quad (14)$$

is the short-ranged half-shell  $t$ -matrix satisfying Eq. (7). For our analysis, and the comparison with coordinate-space calculations, we consider the partial-wave  $S$ -matrix elements, which are obtained from the on-shell  $t$ -matrix elements by the relation

$$S_l(E_{p_0}) = 1 - 2\pi i \mu p_0 \langle p_0 | \tau_l^{CN}(E_{p_0}) | p_0 \rangle.$$

Evaluating the separable Coulomb distorted proton-nucleus  $t$ -matrix involves integrals over the proton-nucleus form factor  $h_{l,i}^c(p)$ . If the short range Coulomb potential is omitted, the functional behavior of the proton-nucleus potential is similar to the one of the neutron-nucleus one, and thus the numerical integration can be carried out as discussed in [11]. However, if it is included, the proton-nucleus form factor falls off more slowly as function of momentum. This implies that larger maximum momenta and an increased number of grid points are necessary to obtain a separable representation of the Coulomb distorted proton-nucleus  $t$ -matrix with the same accuracy as the separable representation of the neutron-nucleus  $t$ -matrix.

### III. RESULTS AND DISCUSSION

For studying the quality of the separable representation of  $t$ -matrices for proton-nucleus optical potentials we consider  $p+^{48}\text{Ca}$  and  $p+^{208}\text{Pb}$   $S$ -matrix elements in the range of 0-50 MeV laboratory kinetic energy. We use the CH89 global optical potential [17] and its rank-5 separable representation in all calculations. The same support

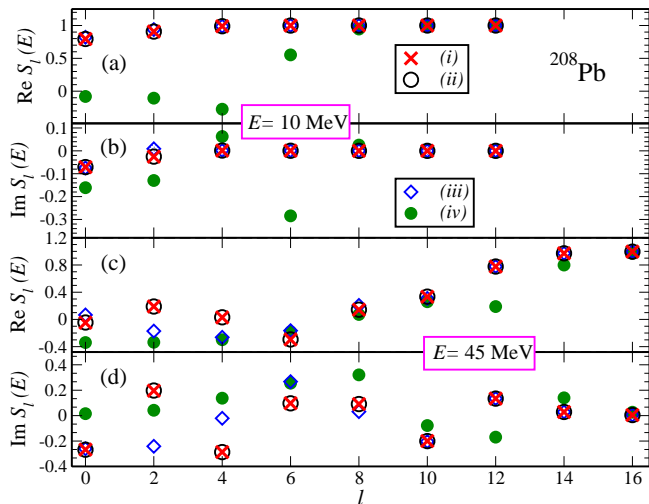


FIG. 1. (Color online) The partial wave  $S$ -matrix for  $p+^{208}\text{Pb}$  elastic scattering obtained from the CH89 [17] global optical potential as function of angular momentum  $j = l + 1/2$ . Panels (a) and (b) show the real and imaginary parts of the  $S$ -matrix at  $E_p = 10$  MeV and panels (c) and (d) provide the same information at  $E_p = 45$  MeV: (i)  $S$ -matrix elements calculated from the separable representation (crosses); (ii) coordinate space calculation (open circles); (iii) the calculation in which the short-range Coulomb potential is omitted (open diamonds) and (iv)  $S$ -matrix elements for  $n+^{208}\text{Pb}$  elastic scattering (filled circles).

points used for the neutron-nucleus separable representation (summarized in Table I of [11]) provide a description of equal quality for the proton-nucleus  $S$ -matrix elements. This is demonstrated for  $p+^{48}\text{Ca}$  scattering at 38 MeV laboratory kinetic energy in Table I, which gives the  $S$ -matrix elements calculated with the separable representation of the Coulomb distorted proton-nucleus  $t$ -matrix, together with the corresponding direct calculations performed either in momentum or coordinate space.

Similar results for the  $p+^{208}\text{Pb}$   $S$ -matrix elements are shown in Fig. 1. The top two panels (a) and (b) show the real and imaginary parts of the  $S$ -matrix elements at 10 MeV laboratory kinetic energy while the bottom two panels (c) and (d) show the real and imaginary parts of the  $S$ -matrix elements at 45 MeV. At 10 MeV the partial-wave series converges much faster, thus we do not show matrix elements beyond  $l = 12$ . First, we note that the momentum space  $S$ -matrix elements calculated with the separable representation (crosses) agree perfectly with the corresponding coordinate-space calculation (open circles).

To illustrate the effects of the short-range Coulomb potential on the  $S$ -matrix elements, we show a calculation in which this term is omitted (open diamonds). As indicated in Fig. 1, only the low  $l$  partial waves are affected. To demonstrate the overall size of all Coulomb effects for  $^{208}\text{Pb}$ , we also plot the corresponding  $n+^{208}\text{Pb}$   $S$ -matrix elements at the same energies (filled circles).

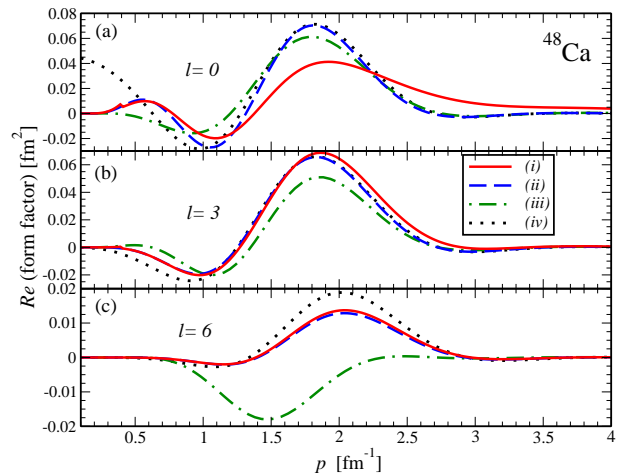


FIG. 2. (Color online) The real parts of the partial wave proton-nucleus form factor for  $^{48}\text{Ca}$  as function of the momentum  $p$  for selected angular momenta  $l$ : (a)  $l = 0$ , (b)  $l = 3$ , and (c)  $l = 6$ . The form factors are calculated at  $E_{c.m.} = 36$  MeV and based on the CH89 global optical potential: full calculations (i) are compared to those omitting the short range Coulomb (ii), the neutron-nucleus form factor (iii) and the Coulomb distorted neutron-nucleus form factor (iv).

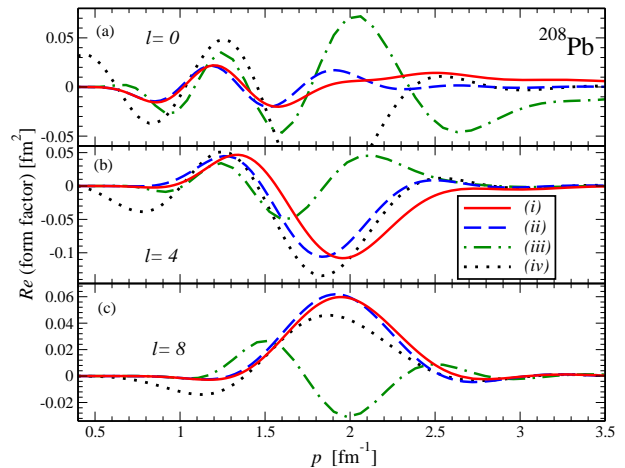


FIG. 3. (Color online). Same as Fig. 2 but for  $^{208}\text{Pb}$ . The form factors for  $l = 0$  (a) and  $l = 4$  are calculated at  $E_{c.m.} = 21$  MeV but for  $l = 8$  (c) these are calculated at  $E_{c.m.} = 36$  MeV.

The differences between the crosses and the filled circles demonstrate the importance of the correct inclusion of the Coulomb interaction.

Next we examine the form factors of the separable representation in detail. In Fig. 2 we compare  $p+^{48}\text{Ca}$  form factors for selected angular momenta calculated with the proton-nucleus potential and the short-range Coulomb potential (i) to those calculated with the proton-nucleus potential alone (ii), as well as the  $n+^{48}\text{Ca}$  (iii). In addi-



tion we show the Coulomb distorted  $n+^{48}\text{Ca}$  form factor (*iv*) obtained with the techniques introduced in [12] to illustrate that a Coulomb distorted neutron-nucleus form factor differs from the corresponding Coulomb distorted proton-nucleus form factor. First, we observe that, with the exception of  $l = 0$ , the form factors already vanish at  $3.5 \text{ fm}^{-1}$ . For  $l = 0$ , comparing the solid and dashed lines, we see that the short-range Coulomb potential significantly modifies the nuclear form factor. The effects of the short-range Coulomb potential quickly decrease as  $l$  increases.

In Fig. 3, we show a similar calculation for the  $^{208}\text{Pb}$  form factors. With the larger charge, the overall observations are maintained but magnified. For  $l = 0$ , the short range Coulomb force creates a very slow fall-off of the proton form factor, and only for  $l = 8$  is the short-range Coulomb potential sufficiently weak to produce a negligible effect on the proton-nucleus form factor. Again we see that for the angular momenta shown, the Coulomb distorted neutron-nucleus form factor does not resemble the Coulomb distorted proton-nucleus form factor, emphasizing the need for a proper introduction of the Coulomb force in the EST scheme.

#### IV. SUMMARY AND CONCLUSIONS

We have generalized the EST scheme [11, 13] so that it can be applied to the scattering of charged particles with a repulsive Coulomb force. To demonstrate the feasibility and accuracy of our method, we applied this Coulomb EST scheme to elastic scattering of  $p+^{48}\text{Ca}$  and  $p+^{208}\text{Pb}$ . We found that the same EST support points employed to obtain the neutron form factors can be used for the separable representation of the proton-nucleus potential. We showed that the momentum-space  $S$ -matrix elements calculated with the separable representation of the Coulomb distorted proton-nucleus po-

tential agree very well with the corresponding coordinate-space calculation. Since changing from a plane wave to a Coulomb basis preserves the time reversal invariance of the separable potential, the separable Coulomb distorted proton-nucleus off-shell  $t$ -matrix also obeys reciprocity.

We also studied the effects of the short-range Coulomb potential on the proton-nucleus form factor. We found that, with the exception of the lowest partial waves ( $l = 0, 1$  for  $^{48}\text{Ca}$  and  $l = 0, 1, 2$  for  $^{208}\text{Pb}$ ), the form factors already vanish at  $3.5 \text{ fm}^{-1}$ . For the lowest partial waves the short range Coulomb force creates a very slow fall-off for the proton-nucleus form factor at high momenta. The effects of the short-range Coulomb potential quickly decrease as  $l$  increases and almost vanish for  $l=6$  ( $^{48}\text{Ca}$ ) and  $l=8$  ( $^{208}\text{Pb}$ ).

In addition, we demonstrated that the proton-nucleus form factor is very different from the Coulomb distorted neutron-nucleus form factor computed according to [12]. Thus, when applying those form factors in a  $A(d,p)B$  Faddeev calculation, it will be mandatory to evaluate neutron and proton-nucleus form factors separately.

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