A hysteresis-like effect for insect control strategies

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An open problem

What would have been a better title for this presentation?

What is a better name for our “hysteresis-like effect”?

Any feedback on this question will be greatly appreciated.

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Controlling the spread of Chagas disease: The motivation for this research

- Chagas disease is a major health problem in rural South and Central America.
    - An estimated 8 to 11 million people are infected.
    - Often fatal; over 10,000 per year attributed to it.
    - Caused by the parasite *T. cruzi*.
    - Transmitted to humans mainly through the bite of insect vectors, *triatomines* aka "kissing bugs."
    - Triatomines colonize poorly built houses and bite at night.
- Insecticide spraying is one of the most widely adopted control measures for Chagas disease.
- We have been studying models of (re)infestation of housing units by insect vectors with the goal of assessing effectiveness of spraying strategies and deriving recommendations.
- The findings appear to be more broadly applicable.

Modeling insecticide control of (re)infestation: General features

- Hosts are housing units.
- m is a constant number of total units (no "demographics").
- Variables:
  - S is the number of units susceptible to infestation.
  - I is the number of infested units.
  - $m - S - I$ is the number of units temporarily protected by insecticide.
- The effect of insecticide wears off over time.
- Infestation may originate from infested units or from a sylvatic reservoir.
- Our models are of type *SIRS* with a reservoir.

Schematic representation of our basic model

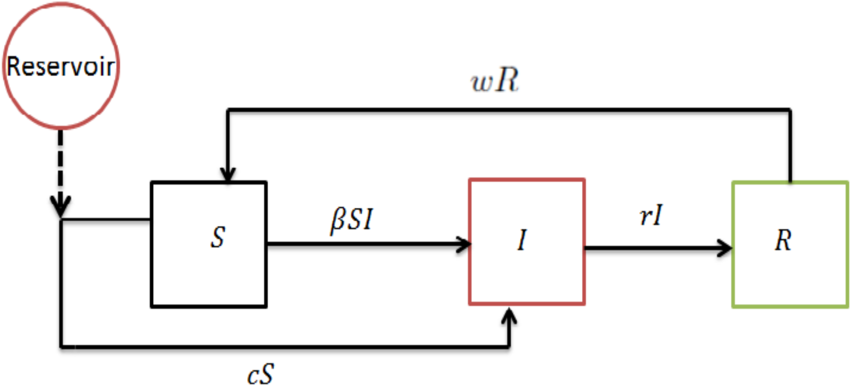

Figure: β—rate of house-to-house infestation, c—rate of infestation from sylavtic areas, w—rate at which insecticide decays, r—insecticide spraying rate. Only infested units get treated in this model.

The basic model

In our basic model, the variables change according to:

$$\frac{dS}{dt} = -\beta IS - cS + w(m - S - I)$$
$$\frac{dI}{dt} = \beta IS + cS - rI.$$

- β—rate of house-to-house infestation,
- c—rate of infestation from sylavtic areas,
- w—rate at which insecticide decays,
- r—insecticide spraying rate.
  This is our control parameter.

In the basic model, only infested units get treated.

Main predictions of the basic model

Theorem

Consider the basic model of the previous slide.

- *When* $c = 0$*:*
  - *When* $\beta m - r \leq 0$*, then the infestation-free equilibrium* $IFE = (S^*, I^*) = (m, 0)$ *is the only biologically feasible equilibrium. It is both locally and globally asymptotically stable.*
  - *When* $\beta m - r > 0$*, then the* $IFE = (S^*, I^*) = (m, 0)$ *is unstable. All trajectories that start with infested units asymptotically approach a second, endemic, equilibrium* EE*.*
- *When* $c > 0$*:*
  - *There exists a unique biologically feasible equilibrium* EE*. It is endemic and both locally and globally asymptotically stable.*

The long-term cost and the budget

We conceptualize the long-term cost of a given spraying strategy as the amount of insecticide that is used over a given long but fixed time interval ΔT.

While there are other monetary costs involved, the amount of insecticide used is closely related to the danger of evolving resistance and side effects due to its toxicity.

When $EE = (S^*, I^*)$, then this cost can be expressed, after suitable scaling, as

$C(r) = r\,I^*(r)$.

When $C(r)$ is bounded from above by the long-term budget and r be the rate at which infestation can be detected,

it would appear that the optimal strategy for keeping I^* as low as possible would be spraying at the maximal rate r that falls within these constraints.

A surprise

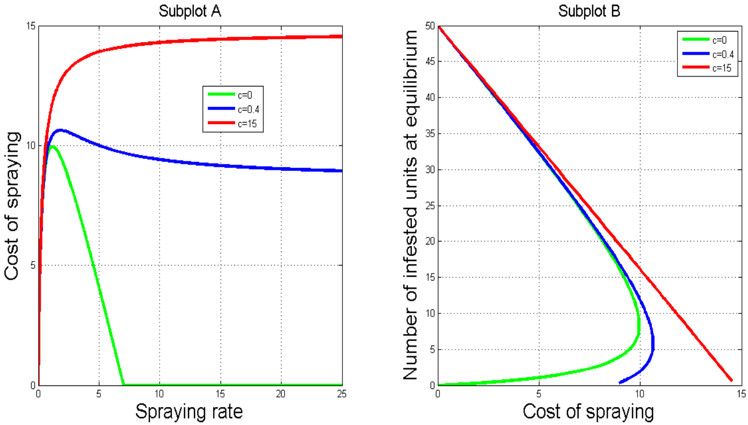

Figure: Spraying rate *vs.* cost in the basic model with $m = 50, \beta = 0.14, w = 0.3$ for selected values of c. When c is sufficiently small, a hysteresis-like effect occurs.

A (hysteresis-like) effect

Definition

Let $I_i^*(r)$ be the numbers of infested units at equilibrium and let $C(r)$ be defined as above. A *(hysteresis-like) effect* occurs if there exist two different spraying rates $r_1 \neq r_2$ such that

(i) $C(r_1) = C(r_2)$,

(ii) $I_i^*(r_1) < I^*(r_2)$.

When such an effect is present, the optimal strategy might involve initially highly aggressive spraying at a cost higher than what is sustainable in the long run,

to drive the infestation to low endemic levels,

and subsequently maintaining these levels at moderate cost.

Why the (hysteresis-like) effect surprised us

The effect does not occur:

- When all units (instead of only the infested ones) are treated with insecticide, so that

$$\frac{dS}{dt} = -\beta IS - cS - rS + w'(m - S - I)$$
$$\frac{dI}{dt} = \beta IS + cS - rI.$$

Here we needed to replace the parameter w with a corresponding smaller parameter w' to account for the possibility of repeatedly treating units while the insecticide remains still effective.

In the above modified model, the cost monotonically increases with the spraying rate.

By comparing the basic with the modified model, we found that spraying only infested units is always more cost-effective.

Why the (hysteresis-like) effect surprised us

The effect does not occur:

- In the linear DE system without unit-to-unit transmission (when $\beta = 0$):

$$\begin{aligned} \frac{dS}{dt} &= -cS - rS + w(m - S - I) \\ \frac{dI}{dt} &= cS - rI. \end{aligned}$$

Why the effect shouldn't have come as a surprise

The effect obviously occurs:

- When $\beta > 0 = c$.
  Initial spraying at a sufficiently high rate will eradicate the infestation, after which the *IFE* can be maintained at no cost with $r = 0$.

Thus it seems at least plausible that the effect might persist even for $c > 0$,

at least for c sufficiently small relative to β.

The (hysteresis-like) effect in the basic model

Theorem

Consider the basic model defined above and assume $\beta, w > 0$.

(a) *The hysteresis-like effect occurs whenever m is sufficiently large relative to the ratio $\frac{c}{\beta}$.*

(b) *The hysteresis-like effect does not occur when $\frac{c}{\beta} \geq m$.*

The effect is fairly robust

We have proved results similar to the theorem on the previous slide

- for a model that allows for the possibility that some insects survive treatment, hide deeply inside the cracks, and re-emerge after the effect of the insecticide wears off.
- for a class of models that allows for several types of units with
  - different infestation rates β_{ij} and c_i, that may reflect properties of the structure like presence of deep cracks or insect screens, as well as topographical features such as proximity to other units or sylvatic areas,
  - different spraying rates r_i that allow us to study different levels of compliance with insecticide treatment.

In the latter models, when $r_i = 0$ for some type of units, the hysteresis-like effect may not occur even when populations sizes are very large. **These findings indicate that increasing the level of compliance can be much more cost-effective than increasing the overall spraying rate.**

Future work: Related models

We are interested in studying occurrence of the (hysteresis-like) effect in other modifications of the basic model:

- *SIRS*-models with a reservoir and demographics (abandoning housing units and building new ones).
- Discrete-time models that would represent periodic insecticide treatment campaigns.

  Preliminary investigations of such models showed that they may exhibit periodic oscillations in addition to steady states. It remains an open question how complex their dynamics can be.
- Network-based models.

  These models may be more realistic as in real villages the number of units can be fairly small and the rate of house-to-house transmission my be strongly influenced by geography or social factors.

Future work: Finding realistic parameters from data

If the (hysteresis-like) effect does apply to the actual (re)infestation dynamics by disease vectors in a given location, one would like to recommend an initially highly aggressive intervention, so that in he long term the equilibrium level of infestation I^* can be maintained at a fairly moderate cost.

However, as the presence of the hysteresis-like effect and the graph of the cost function $C(r)$ crucially depend on the model parameters, it is important to base any such recommendation on reasonably realistic estimates of the parameters from data. In particular, the ratio $\frac{c}{\beta}$ needs to be assessed.

We are working on techniques for disentangling the effects of house-to-house transmission (as represented by β) from the effects of sylvatic transmission (as represented by c.) These investigations are based on analyzing a set of location maps from Ecuador that show which units experienced (re)infestation.