A hysteresis-like effect for insect control strategies

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Controlling the spread of Chagas disease: The motivation for this research

- Chagas disease is a major health problem in rural South and Central America.
  - An estimated 8 to 11 million people are infected.
  - Often fatal; over 10,000 per year attributed to it.
  - Caused by the parasite *T. cruzi*.
  - Transmitted to humans mainly through the bite of insect vectors, *triatomines* aka "kissing bugs."
  - Triatomines colonize poorly built houses and bite at night.
- Insecticide spraying is one of the most widely adopted control measures for Chagas disease.
- We have been studying models of (re)infestation of housing units by insect vectors with the goal of assessing effectiveness of spraying strategies and deriving recommendations.
- The findings may be more broadly applicable.

Modeling insecticide control of (re)infestation: General features

- Hosts are housing units.
- m is a constant number of total units (no "demographics").
- Variables:
  - S is the number of units susceptible to infestation.
  - I is the number of infested units.
  - $m - S - I$ is the number of units temporarily protected by insecticide.
- The effect of insecticide wears off over time.
- Infestation may originate from infested units or from a sylvatic reservoir.
- Our models are of type *SIRS* with a reservoir.

The basic model

In our basic model, the variables change according to:

$$\frac{dS}{dt} = -\beta IS - cS + w(m - S - I)$$
$$\frac{dI}{dt} = \beta IS + cS - rI.$$

- β—rate of house-to-house infestation,
- c—rate of infestation from sylavtic areas,
- w—rate at which insecticide decays,
- r—insecticide spraying rate.

Main predictions of the basic model

Theorem

Consider the basic model of the previous slide.

- *When $c = 0$:*
  - *When $\beta m - r \leq 0$, then the infestation-free equilibrium $IFE = (S^*, I^*) = (m, 0)$ is the only biologically feasible equilibrium. It is both locally and globally asymptotically stable.*
  - *When $\beta m - r > 0$, then the $IFE = (S^*, I^*) = (m, 0)$ is unstable. All trajectories that start with infested units asymptotically approach a second, endemic, equilibrium EE.*
- *When $c > 0$:*
  - *There exists a unique biologically feasible equilibrium EE. It is endemic and both locally and globally asymptotically stable.*

The long-term cost and the budget

We conceptualize the long-term cost of a given spraying strategy as the amount of insecticide that is used over a given long but fixed time interval ΔT.

While there are other monetary costs involved, the amount of insecticide used is closely related to the danger of evolving resistance and side effects due to its toxicity.

When $EE = (S^*, I^*)$, then this cost can be expressed, after suitable scaling, as

$C(r) = r\, I^*(r)$.

When $C(r)$ is bounded from above by the long-term budget and r be the rate at which infestation can be detected,

it would appear that the optimal strategy for keeping I^* as low as possible would be spraying at the maximal rate r that falls within these constraints.

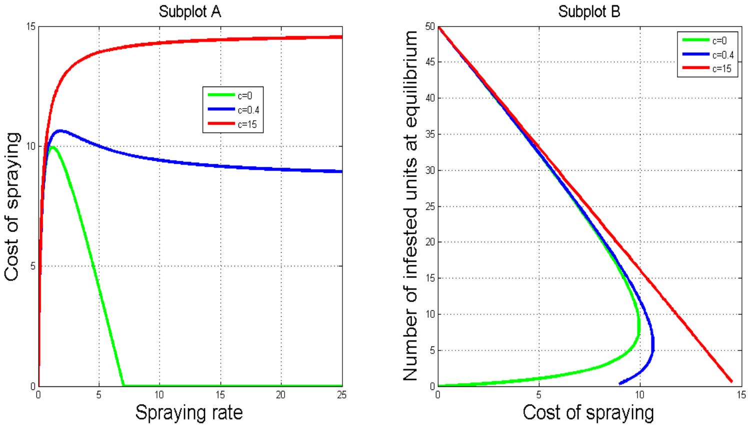

Figure: Spraying rate *vs.* cost in the basic model with $m = 50, \beta = 0.14, w = 0.3$ for selected values of c. When c is sufficiently small, a hysteresis-like effect occurs.

A hysteresis-like effect

Definition

Let $I_i^*(r)$ be the numbers of infested units at equilibrium and let $C(r)$ be defined as above. A *hysteresis-like effect* occurs if there exist two different spraying rates $r_1 \neq r_2$ such that

(i) $C(r_1) = C(r_2)$,

(ii) $I_i^*(r_1) < I^*(r_2)$.

When such an effect is present, the optimal strategy might involve initially highly aggressive spraying at a cost higher than what is sustainable in the long run,

to drive the infestation to low endemic levels,

and subsequently maintaining these levels at moderate cost.

Why the hysteresis-like effect surprised us, and why it shouldn't come as a surprise.

The effect does not occur:

- When all units (instead of only the infested ones) are treated with insecticide.
- In the linear DE system without unit-to-unit transmission (when $\beta = 0$):
  $\frac{dS}{dt} = -cS + w(m - S - I)$
  $\frac{dI}{dt} = cS - rI$.

The effect obviously occurs:

- When $\beta > 0 = c$.
  Initial spraying at a sufficiently high rate will eradicate the infestation, after which the *IFE* can be maintained at no cost with $r = 0$.

The hysteresis-like effect in the basic model

Theorem

Consider the basic model defined above and assume $\beta, w > 0$.

(a) *The hysteresis-like effect occurs whenever m is sufficiently large relative to the ratio $\frac{c}{\beta}$.*

(b) *The hysteresis-like effect does not occur when $\frac{c}{\beta} \geq m$.*

The effect is fairly robust

We have proved results similar to the theorem on the previous slide

- for a model that allows for the possibility that some insects survive treatment, hide deeply inside the cracks, and re-emerge after the effect of the insecticide wears off.
- for a class of models that allows for several types of units with
  - different infestation rates β_{ij} and c_i, that may reflect properties of the structure like presence of deep cracks or insect screens, as well as topographical features such as proximity to other units or sylvatic areas,
  - different spraying rates r_i that allow us to study different levels of compliance with insecticide treatment.

In the latter models, when $r_i = 0$ for some type of units, the hysteresis-like effect may not occur even when populations sizes are very large. These models indicate that increasing the level of compliance can be much more cost-effective than increasing the overall spraying rate.

Future work

- We are interested in studying occurrence of the hysteresis-like effect in other modifications of the basic model:
  - *SIRS*-models with a reservoir and demographics (abandoning housing units and building new ones),
  - discrete-time models that would represent periodic insecticide treatment campaigns,
  - network-based models.
- As the presence of the hysteresis-like effect and the graph of the cost function $C(r)$ crucially depend on the model parameters, it is important to deduce realistic parameters form data. We are working on techniques for disentangling the effects of house-to-house transmission (as represented by β) from the effects of sylvatic transmission (as represented by c.)