Unraveling complex systems: What do brains, the internet, and ant colonies have in common?

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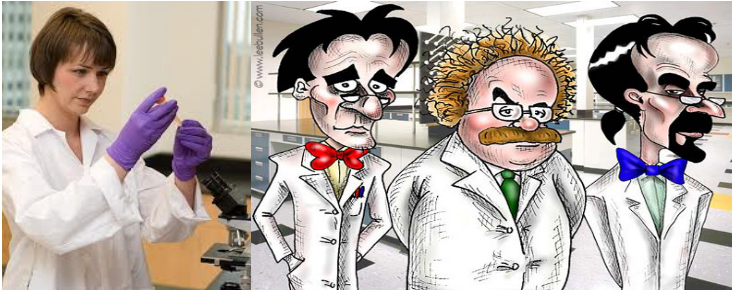
Math Awareness Month April 2011

What are scientists like? A popular view



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Scientists have retained their sense of **wonder.** We marvel at the world around us and we want to **understand** how it works. We like to ask **questions.**
Mathematicians are a little different from other scientists in that we study concepts that are **more** abstract and like answers that give us **absolute certainty.**

Brains

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How can the firing patterns of the neurons *give rise to* **perceptions, feelings, thoughts, and actions?**

The World Wide Web

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How does Wikipedia, a reasonably reliable source of reference, *emerge* **out of the largely uncoordinated creativity of its authors? How does it happen that the accumulating mass of web pages reshapes the way we work, retrieve information, shop, socialize, and spend our leisure time?**

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Ant colonies


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How can an ant colony build elaborate nests and even farm fungi, which amounts to creating a habitat for another species?

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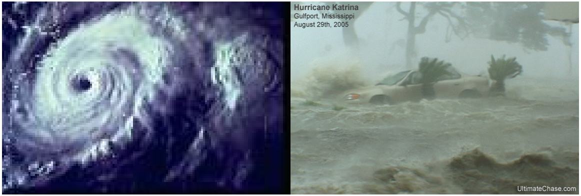
Hurricanes


Hurricane Katrina
Gulfport, Mississippi
August 29th, 2005
UltimateChase.com

Hurricanes



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How do the movements of individual air molecules that are driven by mechanical forces *self-organize* **into powerful winds that can devastate large coastal areas?**

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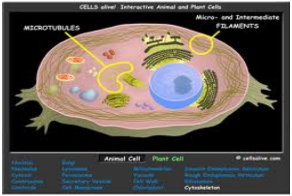
How do the individual decisions ***lead to*** **market equilibrium most of the time and market crashes some of the time? Which mechanisms can prevent systemic failure?**

Cells


CELLS alive! Interactive Animal and Plant Cells
MICROTUBULES
Micro- and Intermediate FILAMENTS
Animal Cell
Plant Cell
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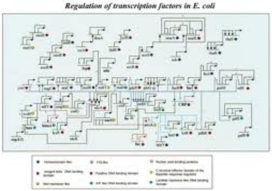
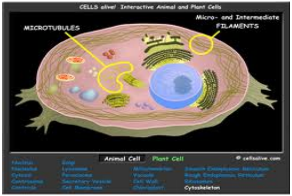
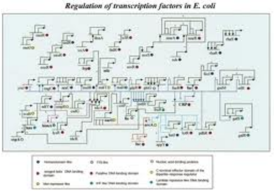
Nucleus
Nucleolus
Cytosol
Centrosome
Centriole
Golgi
Lysosome
Peroxisome
Secretory Vesicle
Cell Membrane
Mitochondrion
Vacuole
Cell Wall
Chloroplast
Smooth Endoplasmic Reticulum
Rough Endoplasmic Reticulum
Ribosomes
Cytoskeleton


Regulation of transcription factors in E. coli
CRP

Cells





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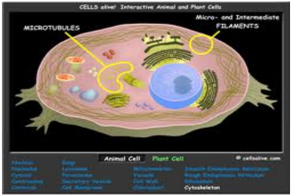

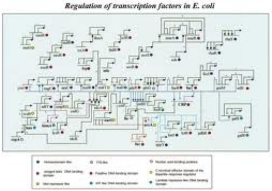

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How does this happen? Why does this work?

Administrative structures

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How does this make institutions function, by and large, the way they are supposed to? When do things go wrong? Which mechanisms can prevent systemic failure?

A pioneer in the study of complex systems

The complex systems we listed are the subject matter of a variety of sciences. Nevertheless, they have a lot in common. The first scientist to clearly discern and spell out these communalities was **Herbert A. Simon (1916-2001)**. He studied administrative structures. And a lot of other things.

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He was not a mathematician, but used a lot of mathematics in his work. And he built on the work of others, including mathematicians Norbert Wiener (1894–1964) and Alfred J. Lotka (1880-1949).

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- Most complex systems have a **hierarchical** and **modular** structure. So there may be more than two levels.
- At the microscopic level agents may behave somewhat randomly and are prone to failure. Despite of this, the behavior of the system at the macroscopic level tends to be fairly robust.

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What makes a Boeing 787 different from our other examples?

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- Note that airplanes are also not very robust to component failure; a fairly common feature of nonadaptive systems.
- All our other examples are complex adaptive systems.
- The distinction is not the same as between "natural" and "engineered" systems. Electrical power grids and transportation systems are complex and fairly adaptive. They are also a lot more robust to component failure than airplanes, but not as resilient as, say, ant colonies.

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- *Theoreticians* construct **models** with the aim of **understanding the mechanisms** by which a system works and **predicting** its behavior.
- Modeling is a **process of selective ignorance.** All models disregard a lot of details. The **art** of modeling is to make the right decisions about what to ignore.

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Fortune cookie: Doing the impossible is kind of fun.

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Mathematics is uniquely suited to unravel the **common features** of complex adaptive systems that we have been marveling at.

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- Study the time courses (trajectories) of the variables either by simulations or mathematical deductions (proofs).
- **Stochastic models** allow for some randomness in the trajectories, **deterministic models** assume that the current state uniquely determines the trajectory.
- **Continuous** (e.g. ODE, PDE) models assume that time can take any (nonnegative) real values; **discrete time** (e.g. difference equation, Boolean) models assume that time moves in discrete steps, that is, only takes integer values.

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- Try to predict (by simulations or deductions) the trajectories of the macroscopic variables, and, if applicable, the change of the equations for the microscopic variables.

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On the next slide we will give (sort of) a definition that will do for this talk.

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- Beware of anthropomorphisms! Systems really only behave **as if** they had "needs" or "tried to."
- Complex adaptive systems rarely find optimal solutions, only reasonably good ones. In Herbert Simon's terminology, they "satisfice."

Ideal gas. A counterexample?

An ideal gas consists of n moles of particles, which gives $\approx 6n\,10^{23}$ agents who can be characterized by $\approx 36n10^{23}$ microscopic variables and who interact according to the laws of (Newtonian) mechanics. This causes the macroscopic variables P, V, T to obey the law

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Personally, I prefer to look at this example as a limiting case of complex systems. We can learn a lot from this example.

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- Details (of the initial state, of the mechanics of the actual bumpings of particles into each other) **don't matter all that much** in the derivation of (1).
- (1) does not hold with certainty, only with probability very, very close to 1.

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- How about learning calculus through group work?

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Neither evolution nor learning is needed for emergence or self-organization.

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Exhibit A: The ideal gas law.

Exhibit B: Hurricanes

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If certain atmospheric conditions hold, then hurricanes will emerge with high probability.

- This probably hasn't been formulated as a theorem, but meteorologists know as much.
- Alas, while the "Theorem" **explains** hurricanes, it does not allow us to **predict** where, precisely, a given hurricane will make landfall. The actual path is **highly sensitive to initial conditions;** weather is a **chaotic system.**

Exhibit B: Hurricanes

Theorem

If certain atmospheric conditions hold, then hurricanes will emerge with high probability.

- This probably hasn't been formulated as a theorem, but meteorologists know as much.
- Alas, while the "Theorem" **explains** hurricanes, it does not allow us to **predict** where, precisely, a given hurricane will make landfall. The actual path is **highly sensitive to initial conditions;** weather is a **chaotic system.**
- The dynamics of complex systems may or may not be chaotic. Chaotic systems may or may not be complex. Chaos theory is something **very different** from a theory of complex systems.

Exhibit C: Yeast cells

Theorem

If cells respond to fairly general feedback mechanisms, then with substantial probability yeast cultures at certain cell densities in a well-stirred vat will spontaneously break up into several clusters that are near-synchronized for cell-cycle state and we will observe corresponding oscillations of oxygen levels.

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Results along these lines are being obtained in an ongoing NIH-funded research project of Prof. T. R. Young that involves Prof. S. Aizicovici, graduate students R. Buckalew, G. Moses, M. Smith, D. Sturgill, and undergraduate students N. Burk from the OU Math Department as well as several external collaborators.

Exhibit D: ODE vs. Boolean dynamics

Theorem

If the ODEs for the agents satisfy certain conditions and their interactions involve certain kinds of intermediaries, then the ODE dynamics precisely matches the discrete dynamics of a Boolean model.

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  D. Terman, S. Ahn, X. Wang, and W. Just; Reducing neuronal networks to discrete dynamics. *Physica D* **237(3)** (2008) 324-338.
- A current joint research project at the OU Math Department of W. Just, B. Elbert, M. Korb, B. Oduro, and T. R. Young aims at proving more theorems of this kind.

Mathematics to learn

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- Differential equations: ODEs, PDEs, differential delay equations
- Stochastic differential equations
- Stochastic processes
- More generally: dynamical systems (discrete, continuous, deterministic, stochastic)
- Theory of computation
- Finite automata
- Probability and statistics
- Statistical mechanics
- ... (you name it)

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  A candidate may be a branch of dynamical systems theory devoted to systems that adapt over time.

Acknowledgement

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