Mathematical models of triatomine reinfestation: What can we learn from them?

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What are we trying to do?

- We want to use mathematical models for answering specific biological questions.
- The questions considered in this talk will focus on insecticide spraying as a control measures for infestation of housing units by triatomines.
- Our models will ignore the actual transmission of Chagas disease by these insect vectors.

Mathematical models

- Mathematical models are greatly simplified versions of reality. They are built by selecting certain aspects of the real world that seem most relevant to answering a particular question.
- This often allows us to predict what would happen in the real world if one makes certain choices; for example, choices of how to implement certain control measures.
- One can then compare the predicted outcomes and implement the most effective course of action.

Model tractability *vs.* biological realism

- There is a tradeoff between making a model simpler and more tractable and making it more realistic but more complicated.
- Simpler, more tractable models tend to make it easier to derive predictions and gain important insights.
- They may occasionally give wrong predictions though if they ignore some important aspects of the real-world situation.
- One can sometimes increase confidence in the predictions of simple models by showing that their predictions remain robust under inclusion of more realistic details.
- Complete validation of the predictions of mathematical models requires experimental verification.

Ingredients of our basic model of insecticide control

- $I = I(t)$ is the number of infested units,
- m is the total number of housing units,
- $S = S(t)$ is the number of susceptible units,
- $R = R(t)$ is the number of recently treated units that are temporarily resistant to (re)-infestation,
- w is the rate at which temporarily resistant units become susceptible again as the effect of the insecticide wears off,
- β is the rate of house-to-house infestation,
- c is the rate of infestation from sylvatic areas.

Some questions about insecticide control

WJ: Here is a list of my most important ones:

Question 1: Is it possible to permanently eradicate triatomines in a given community with insecticide spraying?
If so, how aggressively do we need to spray to achieve this goal?

Question 2: Is it more cost-effective in the long run to spray only infested units, or more cost-effective to spray all units?

Question 3: Is it more cost-effective in the long run to spray very aggressively with higher initial cost or less aggressively with a cost that stays fixed at all times?

Question 4: Is it more cost-effective to spray at fixed time intervals, or to spray as soon as infestation is detected?

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- w is the rate at which temporarily resistant units become susceptible again as the effect of the insecticide wears off,
- β is the rate of house-to-house infestation,
- c is the rate of infestation from sylvatic areas,
- time t can take any real values,
- the spraying rate r is our control parameter.

Schematic representation of our basic model

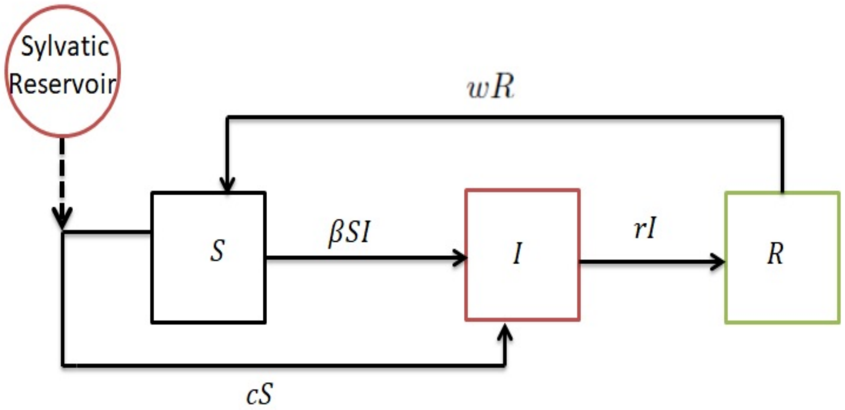

Figure: β—rate of house-to-house infestation, c—rate of infestation from sylavtic areas, w—rate at which insecticide decays, r—insecticide spraying rate. Only infested units get treated in this model.

The basic model

In our basic model, the variables change according to:

$$\frac{dS}{dt} = -\beta IS - cS + wR$$
$$\frac{dI}{dt} = \beta IS + cS - rI$$
$$\frac{dR}{dt} = -wR$$

- w—rate at which insecticide decays,
- β—rate of house-to-house infestation,
- c—rate of infestation from sylavtic areas,
- r—insecticide spraying rate.
  This is our control parameter.

In the basic model, only infested units get treated.

Solutions of the ODEs and equilibria

- The model is a system of three ordinary differential equations (ODEs).
- The solutions or trajectories of the systems are the possible ways in which the three variables $S(t)$, $I(t)$, $R(t)$ could change over time.
- Equilibria (S^*, I^*, R^*) are solutions where these variables remain fixed over time so that we have $(S(t), I(t), R(t)) = (S^*, I^*, R^*)$ at all times t.
- An equilibrium is infestation-free if $I^* = 0$ and endemic if $I^* > 0$.
- A given equilibrium can be approached over time only when it is asymptotically stable.

Theorems on the equilibria of the basic model

When $c = 0$, the model predicts the following:

- When $r \geq \beta m$, then the infestation-free equilibrium $IFE = (S^*, I^*, R^*) = (m, 0, 0)$ is the only biologically feasible equilibrium. It is asymptotically stable.
- When $r < \beta m$, then both the IFE and an endemic equilibrium EE exist. All trajectories that start with infested units asymptotically approach the EE.

When $c > 0$, the model predicts the following:

- There exists a unique biologically feasible equilibrium EE. It is endemic and asymptotically stable.

Without migration of triatomines from sylvatic areas (when $c = 0$), eradication will be achieved with sufficiently high spraying rates. When migration of triatomines from sylvatic areas does occur ($c > 0$), insecticide treatment alone will not achieve eradication.

How about cost?

In the long run the system will be very near an equilibrium with infestation level I^*.

If we spray only infested units, then this cost can be expressed as $C(r) = r\, I^*(r)$.

If we spray all units, then this cost can be expressed as $C_{all}(r) = r\, m$.

When we want to keep the endemic equilibrium level of infestation level at I^* with spraying all units, we only need a smaller spraying rate $r^- < r$ than for spraying only infested units.

But our theorem implies that we still have $C(r) = r\, I^* < r^-\, m = C_{all}(r^-)$.

A dual-rate effect

Let's assume we have a certain budget that allows us to only maintain an equilibrium level of infestation I^* that will keep the long-term cost $C(r) = r\, I^*$ within budget.

When β is sufficiently large relative to c, then a dual-rate effect occurs and there are two different spraying rates $r_1 > r_2$ that give different equilibrium levels $I^*(r_1) < I^*(r_2)$ of endemic infestation, while $C(r_1) = r_1\, I^*(r_1) = r_2\, I^*(r_2) = C(r_2)$.

It would then be more effective in the long run to spray at the higher rate r_1, as long as it is feasible to pay a higher cost over an initial period.

The dual-rate effect is fairly robust

The dual-rate effect also occurs:

- In a model that accounts for the possibility that insecticide treatment is not always 100% successful.

- In a model that accounts for heterogeneities in housing units, such as variable distance from each other and to sylvatic areas, how well they are constructed, or whether the owner is reluctant to allow insecticide treatment.

  The latter model suggests that reluctance to allow spraying has a larger potentially detrimental effect than other types of heterogeneities.

What other possible extensions of our basic model would you suggest?

References

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