

A COMPUTER SIMULATION OF STIRLING  
CYCLE MACHINES

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DECLARATION

I, Israel Urieli, hereby declare that this thesis is my own work and that the material herein has not been submitted by me for degree purposes at any other University.

A handwritten signature in black ink, appearing to read 'Israel Urieli', written over a horizontal line.

## ABSTRACT

This thesis describes the development of a computer program to accurately simulate the performance and detailed behaviour of Stirling cycle machines. The program can be used both as a development tool to predict the performance characteristics of particular machines, as well as a research tool to study the inter-related gas dynamic, thermodynamic and heat transfer behaviour of such machines.

Other simulation methods published to date are based on oversimplified pressure drop, friction and heat transfer relationships.

The approach adopted in the present study was to subdivide the machine into a finite number of one-dimensional cells. Complete differential equations of continuity, momentum and energy of the working gas, as well as energy of the regenerator matrix and heat exchanger walls, are developed. In particular the energy equation of the working gas also includes kinetic energy terms whilst the momentum equation includes the effects of working gas acceleration.

The resulting set of non-linear partial differential equations is solved numerically by the 'method of lines', due regard being taken of the local instantaneous values of dynamic viscosity, Reynolds number, friction factor and heat transfer coefficient, which are all non-linear empirical functions of the system geometry and fluid properties.

A unique method of accelerating convergence of the solution to cyclic steady state is used. The effect of the number of cells of the system subdivision on the accuracy and consistency of the results has also been investigated.

Simulation results of a hypothetical test engine are presented. These include, inter alia, efficiency and indicated power versus rotational speed using air, helium and hydrogen as the working gas. Three-dimensional plots showing incremental temperature, flow and pressure profiles through a complete cycle are presented. The results show the detailed behaviour of the working fluid as influenced by the various machine parameters and working fluid properties, and as such help to provide a new insight into the complex behaviour of Stirling cycle machines.

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