

CARTESIAN CONTROL OF VGT MANIPULATORS APPLIED TO DOE HARDWARE

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ABSTRACT

This paper introduces a novel method for Cartesian trajectory and performance optimization control of kinematically-redundant truss-based manipulators (TBMs): *The Virtual Serial Manipulator Approach*. The approach is to model complex parallel-actuated TBMs as simpler kinematically-equivalent virtual serial manipulators. Standard control methods for kinematically-redundant serial manipulators can then be adapted to the real-time control of TBMs. The forward kinematics transformation can be calculated more efficiently using the equivalent virtual parameters, compared to the computationally intensive parallel-actuated forward kinematics transformation. The method is applicable to any TBM whose modules can be modeled as a virtual serial chain. It also handles TBMs constructed of dissimilar modules, and compound manipulators with serial and parallel-actuated joints. The method is applicable for any level of kinematic redundancy. The method is applied to Cartesian control of the Selective Equipment Removal System deployment manipulator, an eight-dof manipulator proposed for the DOE Decontamination & Dismantling (D&D) program.

1. INTRODUCTION

Truss-Based Manipulators (TBMs, also referred to as Variable Geometry Truss Manipulators, VGTMs) are statically determinate trusses where some of the members are linear actuators, enabling the truss to articulate. Such

devices have been proposed for a variety of tasks, including remote nuclear waste remediation (Salerno and Reinholtz, 1994, and Stoughton et.al., 1995) and space cranes (Chen and Wada, 1990). The following are characteristics of TBMs that are potential improvements over the state-of-the-art in large serial manipulators. When properly designed, all TBM members are loaded axially, thus increasing stiffness and load bearing capability with a lightweight structure. They are modular, with kinematically redundant degrees-of-freedom (dof). The redundancy can be used to optimize performance, including snake-like motion to avoid obstacles. A TBM has an open structure allowing routing of cables hoses, and other utilities.

Other authors have worked in the area of Cartesian control of kinematically-redundant TBMs. Several groups of researchers have proposed use of a "backbone curve" to resolve the redundancy of these manipulators. Salerno(1989) uses parametric curves to place the intermediate links of VGTMs, and the solution is achieved by closed-form relationships. Chirikjian and Burdick (1991) use the backbone curve for the inverse kinematics of modular extensible hyper-redundant manipulators. They formulate the algorithm in a manner suitable for parallel computation. Naccarato and Hughes (1991) compare the backbone curve method to a more "traditional" approach to resolving the inverse kinematics for VGTMs. They find reduced real-time computations using the backbone curve method. The backbone curve method is attractive due to low computation and obstacle avoidance, but the method does not admit optimization of

other performance criteria such as joint limit avoidance and singularity avoidance.

Salerno (1993) solved the inverse kinematics problem for hyper-redundant VGTMs using the pseudoinverse of the Jacobian matrix and projection of objective function gradients into the Jacobian null-space to achieve performance optimization. Due to the parallel-actuated complexity of VGTMs, the Jacobian matrix was derived by numerical differentiation at each control step. Generally, numerical differentiation is to be avoided in digital control applications. However, this work demonstrated that it is possible to control a hyper-redundant manipulator (thirty-dof) via the pseudoinverse in real-time, using a PC-compatible computer. Previous authors have stated that the pseudoinverse is too slow for real-time control; improvements in computer technology are reversing this. Recent work (Canfield et.al., 1996) numerically calculates the Jacobian matrix for any VGT structure with any actuation scheme.

Two groups of researchers have viewed non-kinematically-redundant VGTMs as equivalent serial manipulators. Subramaniam and Kramer (1992) have solved the inverse position kinematics problem for a six-dof tetrahedron VGT analytically by modeling the device as an equivalent manipulator of six revolute joints. Padmanabhan, et. al., (1992a) have analytically solved the inverse position kinematics problem for the quadruple-octahedral VGT by modeling it as a series of two extensible gimbals.

The current paper presents a novel method for simultaneous trajectory and performance optimization control of kinematically-redundant TBMs. The theory has been previously presented (Williams and Mayhew, 1996). The approach is to model complex parallel-actuated TBMs as kinematically-equivalent virtual serial manipulators. A virtual-to-real manipulator inverse mapping is required, but this is accomplished module by module rather than for the entire manipulator. With this paradigm, standard control methods for kinematically-redundant serial manipulators can be adapted to the real-time control of TBMs. The pseudoinverse of the virtual serial manipulator Jacobian matrix (derived analytically) is used, with objective function gradient projection into the null-space for performance optimization.

The method is applicable to any TBM whose modules have a virtual serial model (Williams, 1995). The method is also applicable for any level of kinematic redundancy. Compound manipulators constructed of serial and parallel joints are controlled naturally by this method.

This paper is organized as follows. The next section presents general theory. The following section applies the general theory to the DOE eight-dof Selective Equipment Removal System deployment manipulator. The last major section presents computer simulation and hardware control for this device.

2. VIRTUAL SERIAL MANIPULATOR CONTROL

2.1 "Traditional" Approach

Simultaneous trajectory following and performance optimization is obtained for kinematically-redundant serial manipulators via the well-known resolved-rate algorithm (Whitney, 1969, and Liegeois, 1977):

$$\dot{\Theta} = J^+ \dot{X} + (I - J^+ J)z \quad (1)$$

$\dot{\Theta}$ is the required vector of joint rates; $J^+ = J^T (JJ^T)^{-1}$ is the Moore-Penrose pseudoinverse of the manipulator Jacobian matrix; \dot{X} is the commanded Cartesian trajectory, leading to the particular solution for joint rates; and $(I - J^+ J)$ is the matrix projecting an arbitrary vector z into the null-space of the Jacobian matrix, known as the homogeneous solution. If the arbitrary vector is defined as $z = k \nabla H(\Theta)$, a user-defined objective function (or combination of functions) $H(\Theta)$ can be optimized, where k is an appropriate gain (see Williams, 1994a, for $H(\Theta)$ definitions for joint limit avoidance, singularity avoidance, and obstacle avoidance).

The above discussion relates to serial kinematically redundant manipulators. Figure 1 shows a TBM with four three-dof modules (twelve-dof) to position and orient the end member in the three planar Cartesian freedoms x, y, θ . Each module has rigid member L_0 and actuators L_{1i}, L_{2i}, L_{3i} .

A possible Cartesian coordination algorithm for the manipulator of Fig. 1 adapted from Eq. 1 is given below:

$$\dot{L} = J^+ \dot{X} + (I - J^+ J)z \quad (2)$$

where \dot{L} is the vector of twelve linear actuator rates, and J is the Jacobian matrix mapping the linear actuator rates to the Cartesian rates of the end-effector, \dot{X} . A possible implementation of this controller is shown in Fig. 2. The total actuator rates from Eq. 2 are integrated to commanded actuator lengths (assuming the actuators have position and not rate feedback). The feedback signal for the servo controller is measured linear actuator length (twelve total).

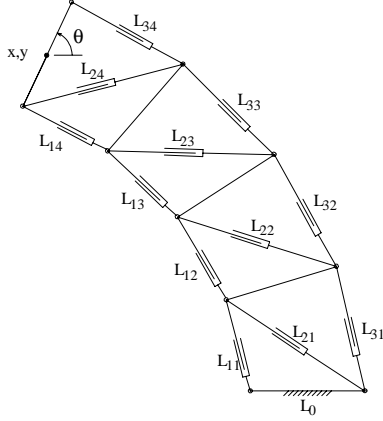


Figure 1. Twelve-dof Planar TBM

The problem with the control concept of Eq. 2 and Fig. 2 is that the Jacobian matrix J is difficult to determine symbolically, due to the complexity of parallel-actuated modules compared to serial manipulator chains. For the planar case, the problem is tractable, but the complexity significantly increases for manipulators constructed from spatial modules. For example, Salerno (1993) simulates Eq. 2 for control of a hyper-redundant manipulator constructed of spatial active truss modules. To determine the Jacobian matrix J at each control step, numerical differentiation is used. In general, numerical differentiation is not robust for digital control applications.

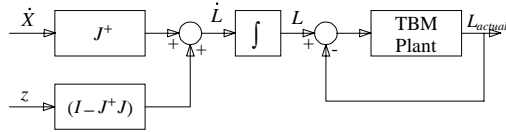


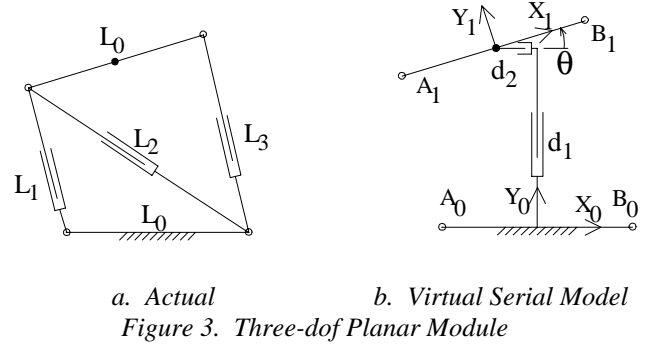
Figure 2. "Traditional" Controller

2.2 Virtual Serial Manipulator Approach

The current paper presents a control algorithm for simultaneous Cartesian trajectory control and performance optimization of TBMs. The crux of the idea is to replace the complexity of the parallel-actuated modules with relatively simpler, kinematically-equivalent, virtual serial manipulator chains. This concept is the Virtual Serial Manipulator Approach.

To illustrate the approach, again consider the proposed TBM in Fig. 1, constructed of the active parallel module of Fig. 3a. In this module, the three actuators position the center of the moving member and orient the moving member with respect to the base. A serial model for this module is two linear actuators d_1, d_2 to control the translation of the moving member center and a revolute joint θ to orient the moving member with respect to the base. Figure 3b presents this kinematically-

equivalent virtual serial module. Figure 4 shows the virtual serial model for the 12-dof planar TBM in Fig. 1, obtained by combining the Fig. 3b model in series. More detail on this example is presented in Williams and Mayhew (1996).



a. Actual b. Virtual Serial Model
Figure 3. Three-dof Planar Module

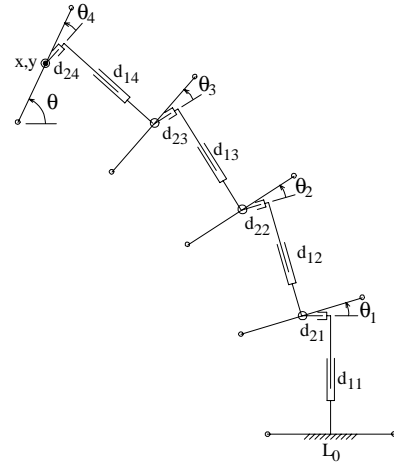


Figure 4. Twelve-dof Planar TBM Virtual Serial Model

The Virtual Serial Manipulator Approach models a modular, parallel, TBM as a virtual serial manipulator which provides kinematically-equivalent motion. The equation for trajectory following and performance optimization is adapted from Eq. 1:

$$\dot{\Phi} = J_V^+ \dot{X} + (I - J_V^+ J_V) z \quad (3)$$

The virtual serial manipulator Jacobian matrix J_V in Eq. 3 is generally easier to determine symbolically than the TBM Jacobian matrix J in Eq. 2, but provides the same motion for the end-effector.

Figure 5 presents the block diagram for controlling a TBM using a virtual serial manipulator model. The control flow is similar to Fig. 2. The difference is that the resolved rate algorithm (both particular and homogeneous solutions) is calculated for the virtual serial manipulator, not the real parallel TBM. The resulting virtual joint rates

$\dot{\Phi}$ are integrated to virtual joint positions Φ . Φ cannot be commanded to the real manipulator, so a transformation from virtual serial joint positions Φ to real parallel actuator lengths is required. This Module Inverse Kinematics is performed independently for each module i . These transformations could be accomplished simultaneously on i processors for improved real-time control throughput. The parallel-actuated complexity is isolated module by module, which is easier (conceptually and computationally) than treating the entire manipulator in a parallel manner. For the module of Fig. 3, the module inverse kinematics solution has been presented (Williams and Mayhew, 1996).

The method is applicable to any planar or spatial TBM whose modules have a virtual serial model. A survey of active truss modules and their virtual serial models has been completed (Williams, 1995). The method is also applicable for any level of kinematic redundancy, from overconstrained (here not all Cartesian dof can be controlled), to non-redundant, to kinematically-redundant, to hyper-redundant. The Virtual Serial Manipulator Approach also handles TBMs constructed of dissimilar modules, and compound manipulators consisting of serial and parallel-actuated joints (such as the case presented in the next section).

A major advantage of the proposed method is that existing methods for control of kinematically-redundant serial manipulators¹⁵ can be adapted for control of TBMs with parallel-actuated complexity. These techniques are implementable in real-time, even for high degrees of kinematic redundancy.

Another major advantage of the proposed method is that the forward kinematics transformation for a TBM may be achieved with the virtual serial joint positions Φ , and not the real actuator lengths L . The assumption is that TBMs are stiff enough to use the virtual parameters (for which there is no feedback) instead of the actual feedback L . For sensor-based control and position control via the resolved-rate algorithm, the forward kinematics transformation must be calculated at in real-time. It is known that the forward kinematics transformation is generally straight-forward for a serial manipulator, and generally difficult and computationally intensive for parallel-actuated manipulators.

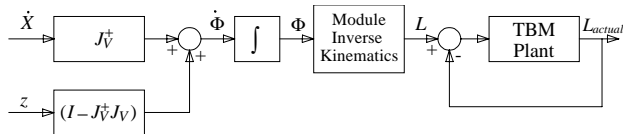


Figure 5. Virtual Serial Approach Controller

3. APPLICATION TO SERS

3.1 SERS Deployment Manipulator

The Selective Equipment Removal System (SERS) was designed for removing radioactive equipment and nuclear materials from nuclear waste sites (Stoughton et.al., 1995). SERS is a spatial manipulator that consists of an autonomous vehicle, a Dual-Arm Work Module (DAWM), and an eight degree-of-freedom deployment manipulator (DM) used to position and orient the DAWM. The DM, shown in Fig. 6, consists of a serial two degree-of-freedom joint, followed by two double octahedral VGT modules connected by a static truss section. The base system consists of two revolute joints; θ_1 can rotate 360° and θ_2 enables the pitch motion from horizontal to vertical. The base VGT module is larger than the tip module and so the static truss section is tapered.

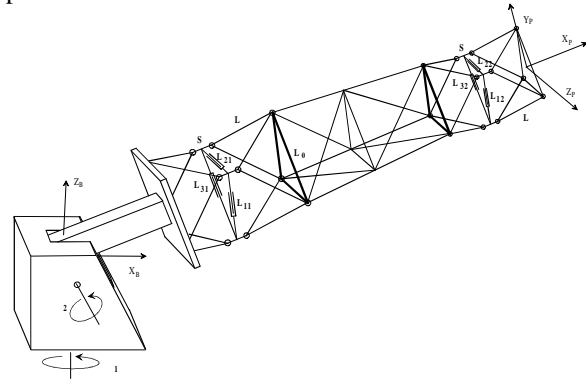
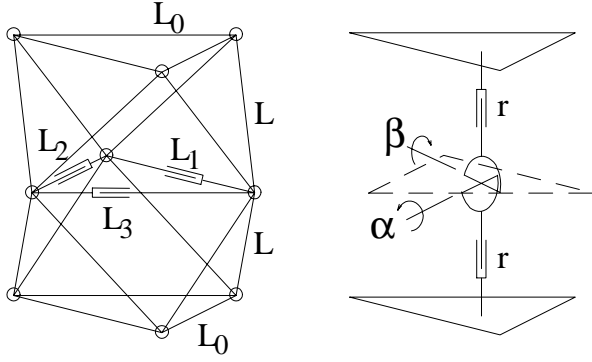


Figure 6. Eight-dof SERS Deployment Manipulator

3.2 SERS Virtual Serial Model

The SERS DM is a compound serial and parallel manipulator. The first two joints of the virtual serial model are identical to those of the real manipulator. The remaining six-dof are comprised of two three-dof double octahedral variable geometry truss modules (Fig. 7a). Each module has three active battens L_1, L_2, L_3 on the mid-plane. The remaining struts L (twelve total) and L_0 (six total) are rigid members.

Figure 7b presents the kinematically-equivalent virtual serial module for the parallel module of Fig. 7a. This is a virtual extensible gimbal, controlling the orientation of the normal to the top plane with α, β rotations about mutually perpendicular axes, plus symmetric, accordion-like extension r of the top plane with respect to the base. This model was first proposed by Padmanabhan et. al. (1992b). A recent study (Williams and Hexter, 1997) presents kinematic design curves for the Fig. 7 module, relating virtual parameter output ranges to design parameters.



a. Actual b. Virtual Serial Model
Figure 7. Double Octahedral VGT Module

Figure 8 presents the kinematically-equivalent virtual serial model for the SERS DM. This model is obtained by using the first two serial joints from the real manipulator, followed by two virtual extensible gimbal models in series, separated by the static truss section. The length from the second revolute joint to the base of the first VGT module is H_1 and the static section length is H_2 . The virtual parameters work together to position and orient the end frame $\{P\}$ with respect to the base frame $\{B\}$. Within the real joint limits θ_{MINi} , θ_{MAXi} , L_{MINi} , and L_{MAXi} , the virtual serial model produces the same motion as the real manipulator.

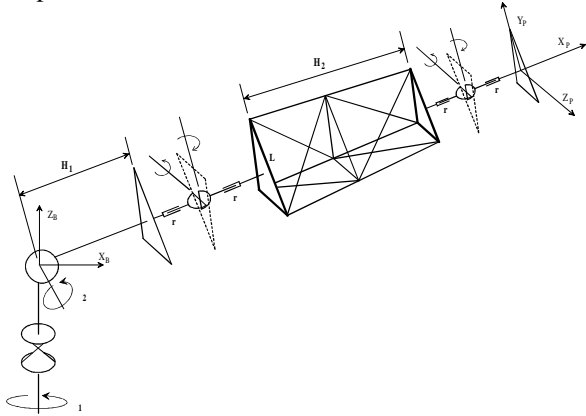


Figure 8. SERS DM Virtual Serial Model

The D-H parameters (Craig, 1989) for the SERS DM virtual serial model in Fig. 8 are given in Table 1. The symbolic Jacobian in base coordinates derived from the D-H parameters was initially a 6x10 matrix, but was simplified to a 5x8 matrix due to two reasons: 1) Columns 3 and 6, 7 and 10 of the Jacobian were added, respectively, because the joint lengths r_i and rates \dot{r}_i for each module are equal due to symmetry. 2) One VGT module allows no rotation about the normal to the truss end-plane, X_p . Two VGT modules in series allow little rotation about X_p . Addition of the 2-dof serial joint

improves this situation, but the X_p rotation is still limited. Therefore ${}^P\omega_x$ (the X component of the angular velocity vector expressed in $\{P\}$) is not controlled and must be eliminated from the velocity equation $\dot{X} = J_V \dot{\Phi}$. This is equivalent to removing the fourth row of ${}^P J_V$ (the virtual Jacobian matrix in $\{P\}$ coordinates).

Since the limited rotation is about X_p but the original Jacobian matrix was derived in the $\{B\}$ frame, a coordinate rotation is required to transform ${}^B J_V$ to ${}^P J_V$. With Eq. 4 a Jacobian can be derived with respect to any control frame $\{F\}$; in this case $\{F\}$ is the base frame $\{B\}$. ${}^P_F R$ is the orthonormal rotation matrix giving the orientation of $\{F\}$ with respect to $\{P\}$.

$${}^P J_V = \begin{bmatrix} {}^P_F R & 0 \\ 0 & {}^P_F R \end{bmatrix} {}^F J_V \quad (4)$$

Equation 4 can also be used for transforming Cartesian rate commands: replace ${}^F J_V$ with ${}^F \dot{X}$. A rate can be commanded in any control frame $\{F\}$, but must to be transformed to frame $\{P\}$ before it is used with ${}^P J_V$ in the resolved-rate algorithm. Terms for the symbolic Jacobian matrix ${}^B J_V$ are given in (Williams, 1996).

Table 1. D-H Parameters for SERS DM Virtual Serial Model

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90	0	0	θ_2
3	0	$H_1 + r_1$	0	0
4	0	0	0	α_1
5	-90	0	0	β_1
6	90	$H_2 + r_1$	0	0
7	0	r_2	0	0
8	0	0	0	α_2
9	-90	0	0	β_2
10	90	r_2	0	0

3.3 Control Algorithm

The specific SERS DM terms for Eq. 3 and the block diagram Fig. 5 are given below.

$$\dot{\Phi} = \{\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\alpha}_1 \quad \dot{\beta}_1 \quad \dot{r}_1 \quad \dot{\alpha}_2 \quad \dot{\beta}_2 \quad \dot{r}_2\}^T \quad (5)$$

$${}^P \dot{X} = \{\dot{x} \quad \dot{y} \quad \dot{z} \quad \omega_x \quad \omega_y \quad \omega_z\}^T \quad (6)$$

$$\Phi = \{\theta_1 \quad \theta_2 \quad \alpha_1 \quad \beta_1 \quad r_1 \quad \alpha_2 \quad \beta_2 \quad r_2\}^T \quad (7)$$

$$z = k\{1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0\}^T \quad (8)$$

$$L = \{L_{11} \quad L_{21} \quad L_{31} \quad L_{12} \quad L_{22} \quad L_{32}\}^T \quad (9)$$

θ_1 and θ_2 of Eq. 7 and L of Eq. 9 are the joint commands that are sent to the controller for positioning and orienting the $\{P\}$ frame with respect to the $\{B\}$ frame in Fig. 6. The base actuation system can also be controlled independently from the resolved-rate algorithm if desired for faster global pointing. θ_1 and θ_2 can be used to move the deployment manipulator into given work area and then the resolved-rate algorithm is used control the entire 8-dof DM within that area. When θ_i , α_i , and β_i are zero, the virtual Jacobian matrix is singular. To avoid the singularity θ_{2MIN} is set to 1° .

The DM has two redundant degrees-of-freedom that enables the option for self-motion or performance optimization. An arbitrary vector of ones was used for the z vector to exercise the manipulator's self-motion in the null-space. Standard objective function gradients may instead be used to optimize performance in terms of joint limit avoidance (requires virtual to real joint limit mapping), singularity avoidance, and obstacle avoidance, among others (Williams, 1994a).

The Module Inverse Kinematics solution is not required for the first two joints because the virtual and actual joints are identical. The Module Inverse Kinematics solution for double octahedral VGT modules has been presented (Williams, 1994b). This solution was implemented in real-time on VGT module hardware, first at NASA Langley Research Center (Williams et al., 1995) and then at Pacific Northwest National Laboratory in Richland Washington.

4. COMPUTER AND HARDWARE SIMULATION

4.1 Computer Simulation

A computer simulation with animation was developed to test the Virtual Serial Manipulator Approach for control of the SERS DM in Fig. 6. This section presents a simulation for the particular solution only, $\dot{\Phi} = J_V^+ \dot{X}$. The homogeneous solution $\dot{\Phi} = (I - J_V^+ J_V)z$ has also been tested. The units are *inches*, *inches/s*, and *rad/s* for length, transitional and rotational velocity, respectively. The DM design values are:

$$H_1 = 64.0 \quad H_2 = 60.0$$

First VGT module:

$$\begin{aligned} L_O &= 36.0 & L_{MAX} &= 55.5 \\ S &= 4.75 & L_{MIN} &= 39.0 \\ L &= 34.0 \end{aligned}$$

Second VGT module:

$$\begin{aligned} L_O &= 27.7 & L_{MAX} &= 39.3 \\ S &= 3.70 & L_{MIN} &= 29.5 \\ L &= 26.3 \end{aligned}$$

The initial joint values are:

$$\theta_1=0^\circ, \theta_2=1^\circ, L = \{45 \quad 45 \quad 45 \quad 34.62 \quad 34.62 \quad 34.62\}^T$$

The corresponding virtual serial joint parameters are:

$$\Phi = \{0 \quad 1 \quad 0 \quad 0 \quad 26.64 \quad 0 \quad 0 \quad 20.72\}^T$$

The Cartesian rate trajectory ${}^B \dot{X} = \{0 \quad 0 \quad 6.5 \quad 0 \quad 0\}^T$ was commanded until a joint limit was reached. First Eq. 4 (with ${}^B \dot{X}$ instead of ${}^B J_V$) was used to determine the required ${}^P \dot{X}$.

Figure 9a shows the initial DM configuration and Fig. 9b presents the final configuration. The following figures also include the real and virtual joint parameters. The parameters T_i , A_i , B_i , and R_i represent virtual joint parameters θ_i , α_i , β_i and r_i , respectively, $i=1,2$.

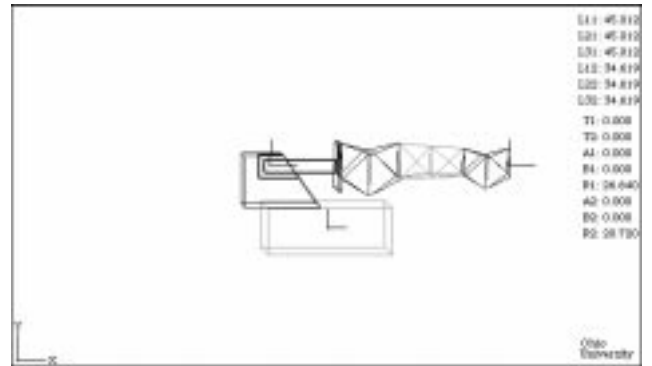


Figure 9a. Initial SERS DM Configuration

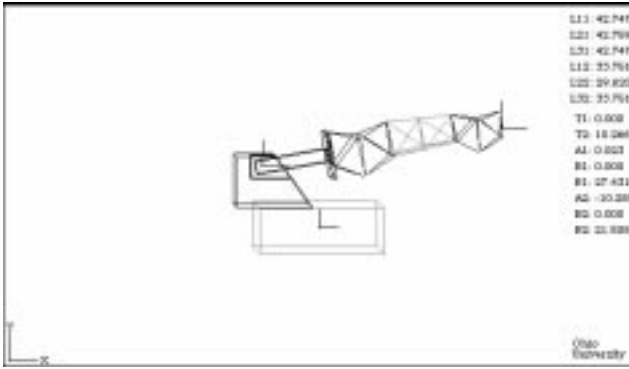


Figure 9b. Final SERS DM Configuration

4.2 Hardware Control

The two three-dof VGT modules and the tapered static truss section (Fig. 6) hardware was built at NASA Langley Research Center and delivered to Pacific Northwest National Laboratory (see Fig. 10). The autonomous vehicle has been built, but the 2-dof serial joint has not yet been built. The theory in this paper was implemented on this hardware for Cartesian rate control of the truss end frame $\{P\}$. The method was demonstrated to be effective in hardware control. Without the two-dof serial joint, the control equations had to be modified (Williams, 1996). The methods of this paper may be extended to handle the moving base. Also, position control may also be achieved using the basic rate control of Fig. 5 (Williams, 1997).



Figure 10. Six-dof, two-VGT SERS DM Hardware

5. CONCLUSION

A novel concept, The Virtual Serial Manipulator Approach, is presented for simultaneous Cartesian trajectory following and performance optimization of truss-based manipulators (TBMs). The approach models complex parallel-actuated TBMs as kinematically-equivalent virtual serial manipulators. A virtual-to-real manipulator inverse mapping is required, accomplished

module by module rather than for the entire manipulator. Existing control methods for kinematically-redundant serial manipulators can then be adapted to the real-time control of TBMs. The pseudoinverse of the virtual serial manipulator Jacobian matrix is used, with objective function gradient projection into the Jacobian null-space for performance optimization. A benefit of the method is that the forward kinematics transformation can be calculated more efficiently using the equivalent virtual parameters, compared to the formidable parallel-actuated forward kinematics transformation.

The method is applicable to any TBM whose modules can be modeled as a virtual serial chain. The Virtual Serial Manipulator Approach also handles TBMs constructed of dissimilar modules, and compound manipulators consisting of serial and parallel-actuated joints. The method is applicable for any level of kinematic redundancy.

The method was applied to the DOE eight-dof SERS DM. The algorithm was developed and a simulation was presented for this case. The method proved to be effective in hardware control of the six-dof portion of the SERS DM which has been built to date.

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