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CAPSTAN AS A MECHANICAL AMPLIFIER

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ABSTRACT

A capstan is simply a cylinder with a flexible body such as a string or cable wrapped around it. Capstans are generally used to assist in the lifting or pulling of heavy objects in the form of winches. By controlling the input tension on the cord, a capstan can dynamically amplify the input. It is hypothesized that a single capstan with multiple cords wrapped about it can act as numerous mechanical amplifiers.

The validity of the classic capstan equation was investigated in this study. It was found that, where the classic equation is able to provide a rough estimation of the dynamics of a capstan system, a more complex equation was needed to fully define the behavior. A more complex equation was developed and compared to the experimental data. It was found to accurately describe the dynamics of the system and explanations were offered to describe the contributing forces.

1. INTRODUCTION

The capstan in conventional terms is used simply whenever a user must lift or pull something which is out of their means to accomplish under their own abilities. Historically they have been used on sailing vessels when heavy sails and anchors required methods other than simple pulleys. Present day capstans are drums driven by electric or hydraulic motors whereas historical capstans were driven under human force. In both situations however the concept is the same; there is a rope wrapped about a drum which is rotated. As force is applied in the direction of rotation, friction is developed between the rope and the drum. The generated friction will then act in the same direction of pull effectively amplifying the users force. An example of a rail mounted capstan can be seen in Figure 1. Capstans are still widely used on sea vessels because of their ability to allow a single or few deck hands to left heavy objects with ease. Other applications range from winches to driving mechanisms in cassette tape players (Figure 2).

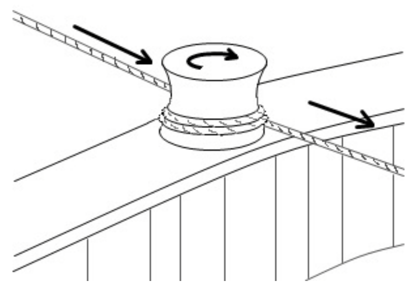

Figure 1 - Rail Mounted Capstan

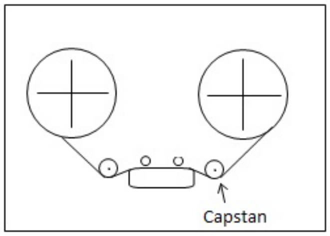

Figure 2 - Cassette Tape Capstan

Generally speaking, the usage of the capstan is to develop sufficient friction force between the rope and the drum to have the rope travel at the same rate as the drum. For most lifting or winching task, any slippage is undesirable. This type of capstan usage could be considered static because the friction between the rope and the capstan is static friction. If the rope slips about the drum however, there is kinetic friction and the system can be considered dynamic.

With both states of friction, the generalized capstan equation (Attaway 1999) determines the tension in both ends of the rope. The capstan equation is shown in Equation 1.

$$T_2 = T_1 e^{\mu\beta} \tag{1}$$

The input tension, the smaller tension historically driven by a human, is represented by T_1 and the output tension, the greater tension used in lifting or winching, is represented by the term T_2. The variable within the exponential are μ, which is the friction coefficient between the drum and the rope and β, the angle in which the rope is in contact with the drum (in radians). Distinction between static and dynamic friction is not taken into account. What the capstan equation is saying, is that if friction and number of wraps are kept constant, a force applied, T_1, will be amplified, T_2, in a linear manner, $e^{\mu\beta}$. Simply stated the capstan is a mechanical amplifier. And electrical analog would be the transistor.

If the human in the conventional capstan usage is replaced by an electrically controlled actuator, the force developed by the actuator could be effectively amplified[1]. Having most of the force transferred to the capstan results in less force being required from the actuator. With respect to conventional cable robotics, which are generally driven by electrical motors, a small motor actuating a cable wrapped about a capstan driven by a large motor would be able to act identically as a single large motor actuating the cable. In the stated system, having the capstan setup would mean added complexity and added weight, large motor plus the small motor. However, if multiple control motor systems are combined with a single drive motor as shown in Figure 3, the large actuating motors in a system may be reduced in both size and weight.

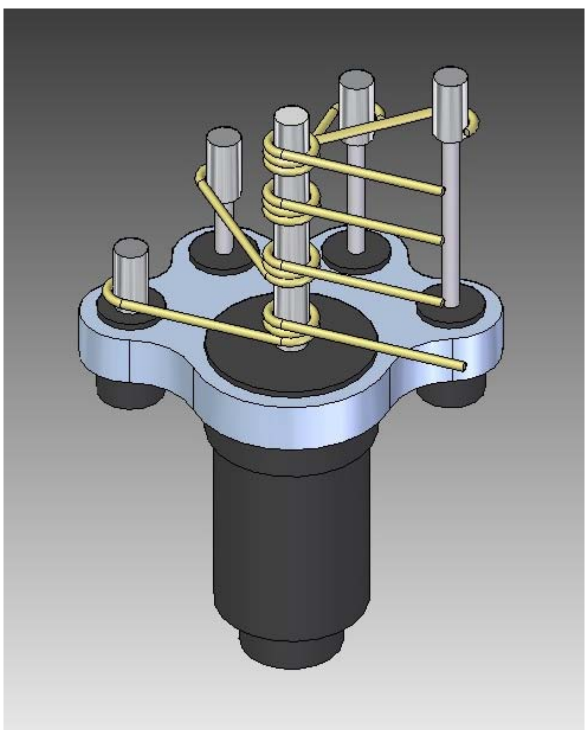

Figure 3 - Parallel Cable Actuator Using Capstan Amplification

[1] T_1 is the end of the rope being pulled in the same direction as the rotation of the capstan drum. If the drum ceases to rotate, T_1 and T_2 switch.

Parallel systems have been implemented in the past (Karbasi, Huissoon and Khajepour 2004) but they operate using clutches. A clutch generally adds significant weight to a system and because of the way that clutches engage, they tend to introduce noise into the system. A parallel capstan system would eliminate the clutches and require simple alterations to an existing robotic system. The system mentioned referenced only electric motors, in actuality any actuator would act the same in the system.

The motivation for the development of the technology was driven by the world of biomimetic robots and prosthetics. Muscle is able to provide high force output for relatively small area. To replicate the force able to be developed by biological systems, large motors or tethered hydraulic or pneumatic systems are necessary. The systems generally add a prohibitively large amount of weight. A parallel capstan system could solve the weight and force problems associated with these biomimetic systems however no system such as this has been implemented as of yet. Aside from the complete parallel system, the dynamics of using the capstan as a mechanical amplifier have not been tested. This study was to investigate the intricacies associate with such a practice.

2. EXPERIMENTAL METHODS

To explore the dynamics of capstan amplification, a horizontal test system was constructed (Figure 4). A cord was wrapped around a cylinder which was driven by a electrical motor. Both ends of the cord were connected to Vernier force sensors to directly measure T_1 and T_2. Direct calibration was conducted to ensure the reading were level. An electric control circuit was designed to pull on one of force sensors to induce a tension into the system. Measurements were taken via an attached Vernier LabPro.

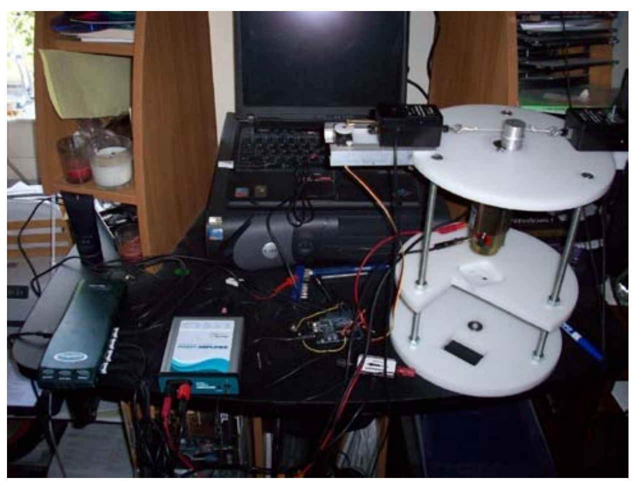

Figure 4 - Test Setup. The three white discs provided the structure for the testing. A motor was mounted on the underside of the top disc and a capstan was attached to the shaft. The two black rectangles are the force sensors and strung between them is the cord. To the left of the left sensor, the servo can be seen which applies the force.

Three raw data sets were collected for each angle measurement: the angle is the angle in which the cord is in contact with the capstan. To account for possible settling or wearing effects, all of the angles were tested once, then the process was repeated two more times. A new section of string was made for each of the tests.

During each test run, a servo motor pulled on the force sensor measuringT_1. The servo stepped up the tension with 10 second intervals. A total of 12 steps were made within each test run. During each step, data was sampled at 20Hz which resulted in 200 data points. The data between 2 seconds and 9 seconds for each of the tests were then averaged to obtain a tension reading for that particular data pair (measurement for T_1 and T_2. These averaged values are what is shown in Figure 5 and Figure 6. Two full sets of data were collected each using a differing diameter capstan. The capstan equation (equation 1) omits any consideration of capstan diameter prompting the need for two sets to determine the validity of the assumption.

To also account for the possible effect of rotational speed, a smaller sample set was created using two different capstan speeds (discussed later). The speed was determined by the voltage supplied to the motor; one data set taken at 10V and another at 20V. The exact speed was not measured however the difference was visually significant and determined by the results to be ultimately irrelevant.

3. RESULTS AND DISCUSSION

An initial observation of the data produced by the tests show well defined trends for each of the angle values (Figure 5 and Figure 6). Each of the trends displays a marked amplification of the input tension, T_1. Equation 1 dictates that as the angle of contact between the string and the capstan increases, so should the amplification, if all else is kept equal. This trend is seen occurring in the data from both of the sets.

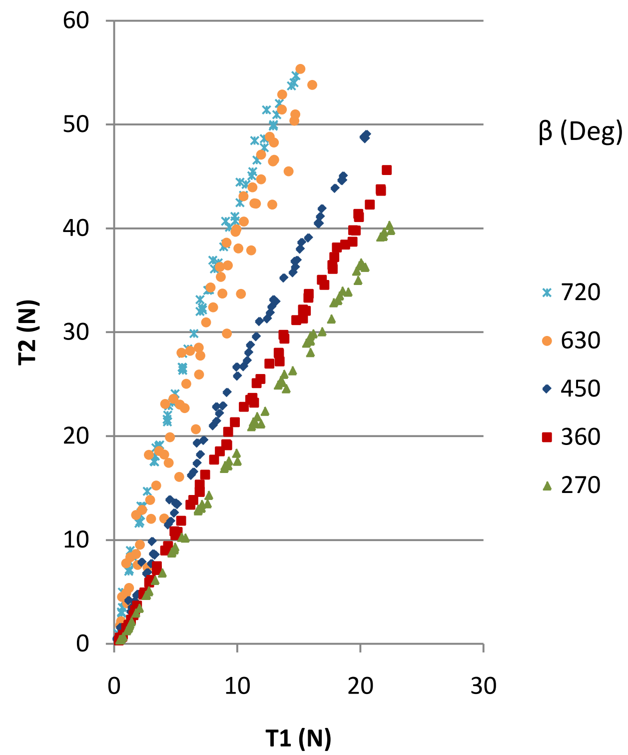

Figure 5 - T2 vs. T1 - 0.5" Capstan Diameter

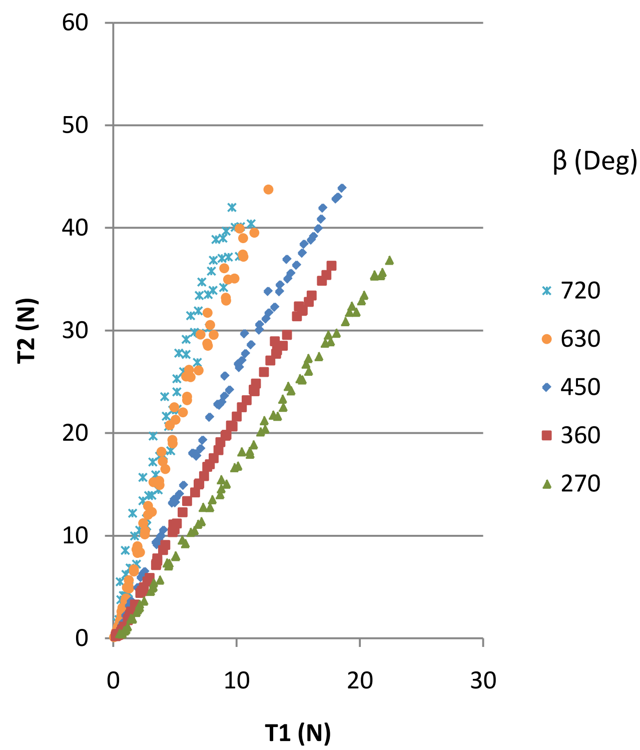

Figure 6 - T2 vs. T1 - 1.0" Capstan Diameter

The two major inherent assumptions that the capstan makes are that diameter and rotational speed have no effect on the behavior of the system. To verify these assumptions the data from both of the tests were compared to each other.

An example of the results comparing capstan diameter is shown in Figure 7. It can be seen that both of the data sets are nearly identical suggesting that diameter has no significant effect on the amplification of the system. Each of the tests produced nearly identical correlations.

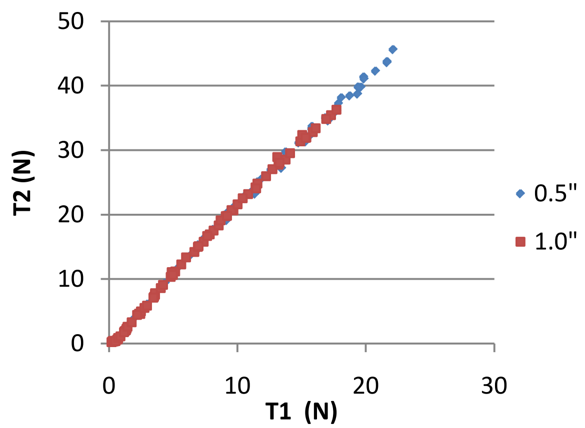

Figure 7 - Diameter Comparison Sample 360 Degrees

Similar results were found in a comparison of the speed influent shown in Figure 8. Each of the angle sets correlated similarly but produced slightly more noisy data than the diameter comparisons.

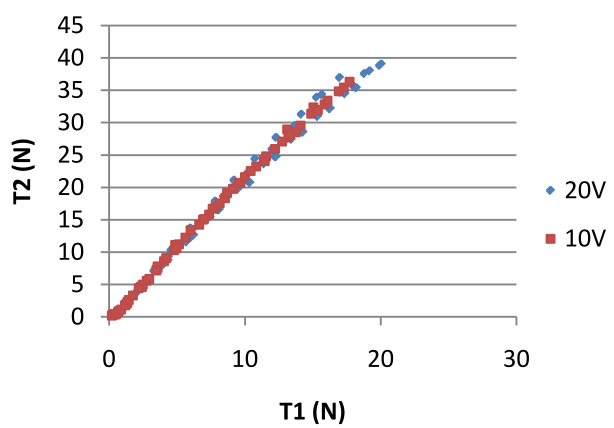

Figure 8 - Rotational Speed Comparison Sample 360 Degrees

Since the two assumptions for diameter and rotational speed were confirmed through testing, further analysis of the data was performed to further determine the adherence of the data to the capstan equation.

Since three of the four variables of the capstan equation are known during testing, the final variable μ, friction, can be used to attempt theoretical fits to the data. The curve fits can be seen in Figure 9 and Figure 10.

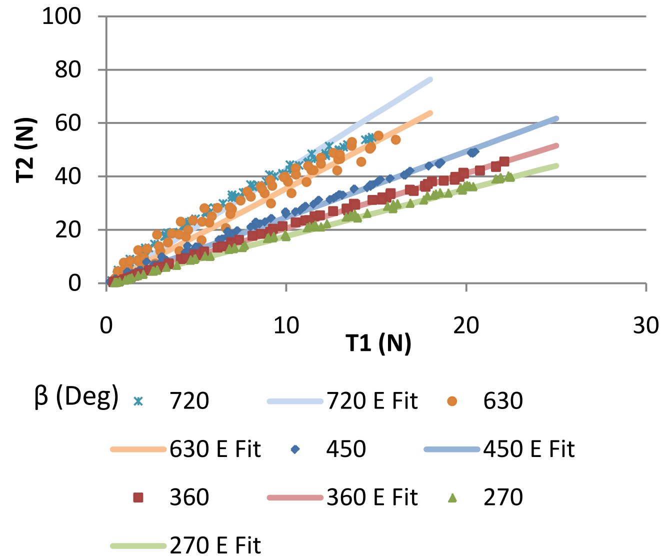

Figure 9 - Capstan equation fitted to the experimental data for the 0.5in diameter capstan

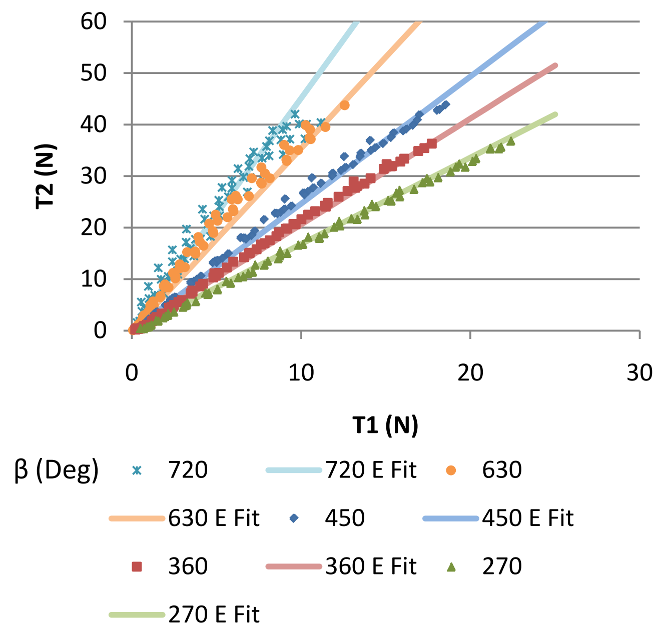

Figure 10 - Capstan Equation Fitted to the 1.0in Diameter Capstan

The lines labeled "E Fit" are the theoretical performance curves at each of the contact angle intervals. The values for the friction variable are summarized in Table 1. More evidence verifying the capstan equation can be interpreted from the nearly identical values found for the value of friction. If the capstan equation is to represent the system, the friction values would need to remain constant across all of the sets of data.

Table 1 - Friction Values for Calculating the Theoretical Performance Curves of the Capstan Equation

Run	0.5"	1.0"
270	0.12	0.11
360	0.115	0.115
450	0.115	0.115
630	0.115	0.115
720	0.115	0.12

The data confirms that a cord being pulled in the direction of rotation of the capstan it is wrapped about will result in amplification of the tension. As the capstan equation stands, given a constant friction and constant angle of contact between the cord and the capstan, there will be a constant amplification of the input tension; defined by the exponential part of Equation 1. As shown in Figure 11 the constant amplification expected from the capstan equation is in fact not constant. More complex behavior is occurring than straight forward amplification.

Going forward, since it has already been shown that there is no difference in the performance between the two diameter capstans, a single graph will be shown when describing system behavior.

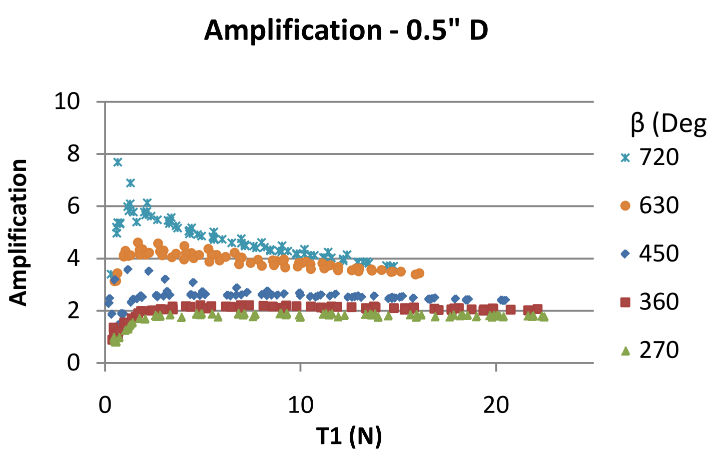

Figure 11 - Amplification Values calculated by comparing the Ratio Between T2 and T1

To understand precisely what behavior is occurring with regards to the amplification, the amplification curve for a contact angle of 360 will be examined (Figure 12).

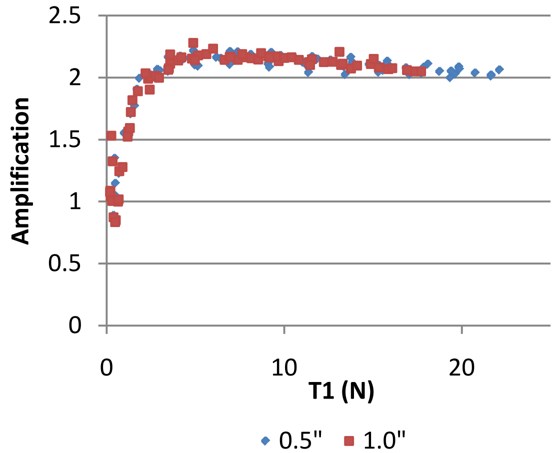

Figure 12 - Amplification data for Contact angle of 360 Degrees for Both the 0.5 inch and 1.0 inch capstans

It is not know specifically what causes the characteristic curve seen in Figure 12 but to mathematically describe the system, the assumption will be made that there are two transient influences leading to a steady state value. The reasoning for these assumptions is the lack of understanding of the physics occurring to cause the data curve and the fact that the data follows the characteristic curve of a double transient.

To better understand the double transient, a simple exponential decay curve is examined against the data in Figure 12. The equation for a basic exponential decay is shown in Equation 2. There is an exponential part which involves an amplification of the exponential by N_0 and the decay dictated by λ. When this characteristic curve is fit to the 360 degree contact angle data (Figure 13) it can be seen that the curve well represents the data greater than 9N but fails to represent it prior to it.

$$f(x) = N_0 e^{-\lambda x} + SS \quad (2)$$

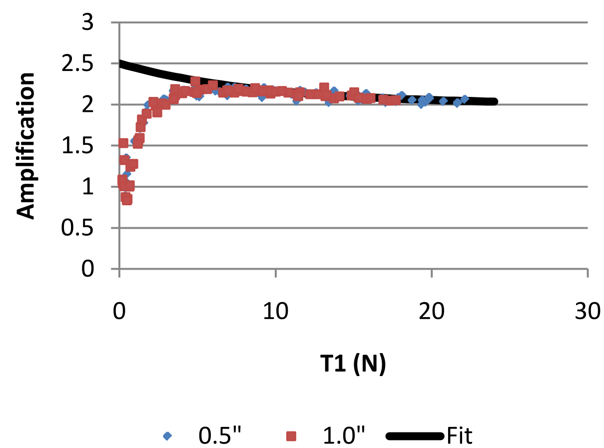

Figure 13 - Exponential Decay function fit to the Model Amplification Data of the 360 degree Contact Angle. The Values used to create the fit curve were N0 = 0.5, lambda = 0.11 and ss = 1.75

The inability for the simple exponential to represent the data prompts the addition of the second exponential to Equation 2. The characteristic curve now becomes Equation 3 shown below.

$$f(x) = -N_d e^{-\lambda_d x} + N_0 e^{-\lambda x} + SS \qquad (11)$$

The additional term with the subscript d represent contribution of the second exponential. The double exponential equation was fitted to the model 360 data and the resulting plot is shown in Figure 14. With the inclusion of the second exponential, the data is now well represented. Values for goodness of fit are shown also on Figure 14.

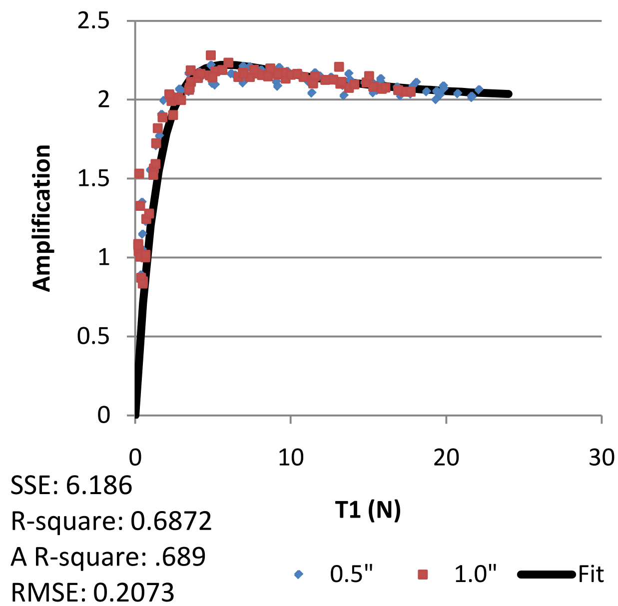

Figure 14 - Double Exponential decay function fit the model amplification data for the 360 degree contact angle. The values used to fit the curve to the data are N0 = 0.5, lambda = 0.11, ss = 2, nd = 2.5 and lambda_d = 0.7.

The double exponential was fitted to each of the data sets and the result is shown in Figure 15. When Figure 15 is compared to Figure 11 it can be seen that each of the curves follow the data which was experimentally measured. The values used to fit the curves are shown in Table 2. The values for the 540 degree contact angle (540 degrees required the two sensors to be in the same spot and was unable to be measured) were estimated by observing trends made by values of each of the variables.

Table 2 - Summary of values used to fit the double exponential decay curve to the experimental data

Angle (deg)	270	360	450	630	720	*540
SS	1.75	2	2.3	3.1	3.4	2.7
No	0.25	0.5	1.0	1.8	2.5	1.3
λ	0.11	0.11	0.11	0.11	0.11	0.11
N_d	2	2.5	3.3	4.9	5.9	4.2
λ_d	0.6	0.7	0.9	1.5	2	1.17

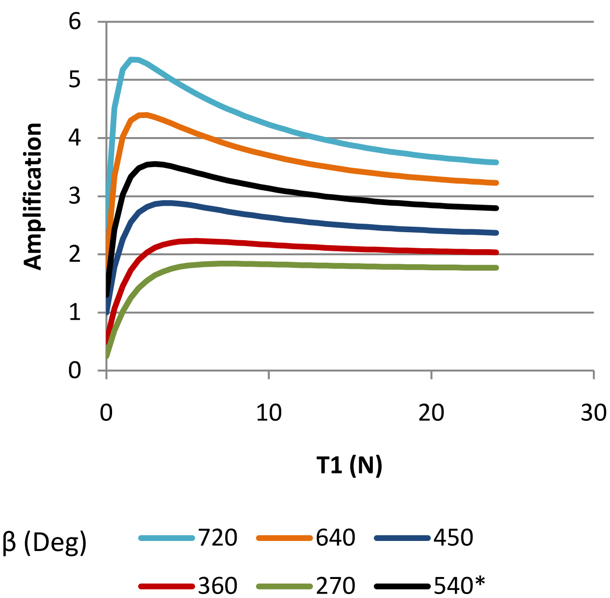

Figure 15 - Theoretical amplification curves as a result of the curve fitting of the double exponential decay function. The 540 degree contact angle was included but could not be measured with the experimental setup

The easiest trend to be seen from the fitted data is the consistent value of 0.11 for lambda. Although speculative, this value may be representative of friction being that it is expected to remain unchanged.

Figure 16 shows the values of the steady state variable. The data appears to trend to a value of 1. Intuitively, if the angle were 0, the input tension would be acting directly on the output resulting in a 1:1 ratio. A straight line was fitted to the data with an intercept at 1 to represent this characteristic.

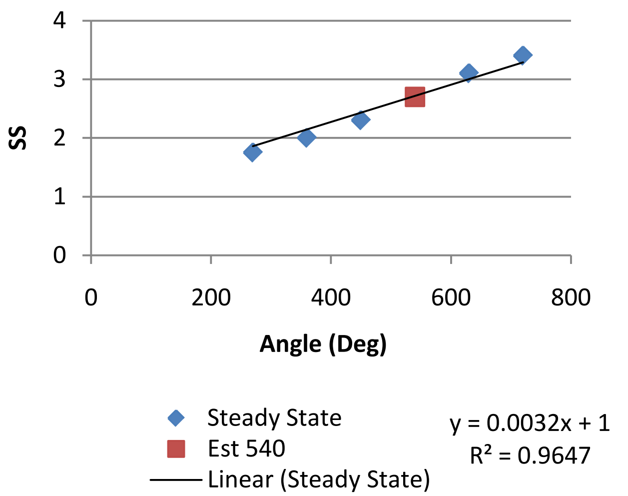

Figure 16 - Comparison of steady state values produced by fitting the double exponential to the experimental data.

The values for N_0 produce less intuitive results but the boundary condition at 0 degrees of contact can still be assumed. When the system has zero contact with the capstan, the input is directly pulling on the output and the exponentials go to zero. A basic power fit was applied to the values only because of the good fit (Figure 17).

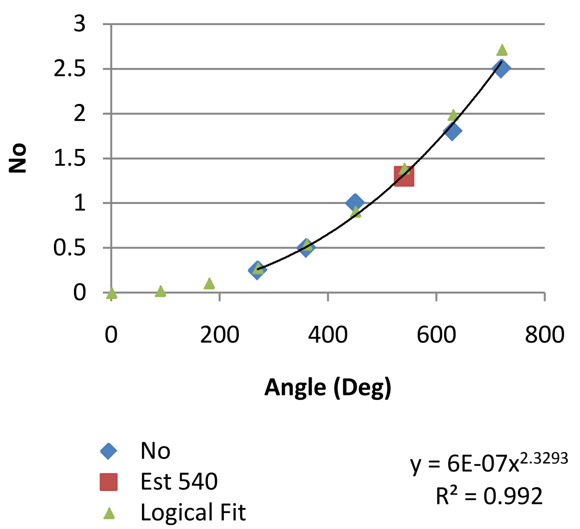

Figure 17 - Comparison of N0 values produced by fitting the double exponential to the experimental data

The values for N_d exhibit the same linear nature as the steady state values (Figure 18) but similarly to N_0, at 0 angle of contact, N_d must be zero. One explanation for what influences the d values is that they are related to settling of the string used during testing. The string used for test was a multi-strand nylon cord. As the angle in which the cord is in contact with the capstan increases, any effect would increase with the length in contact with the capstan which happens to increase linearly. This same path of thinking can also somewhat make sense of λ_d (Figure 19) in the fact that as length increases, if the cord is compressed, area will be expected to increase. The values for λ_d didn't follow any good trends but to arrive at an estimate for 540 degrees, an exponential function was applied to the data.

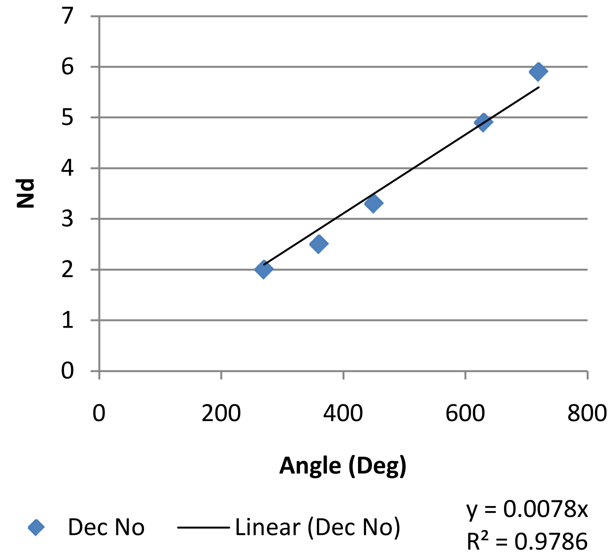

Figure 18 - Comparison of Nd values produced by fitting the double exponential to the experimental data

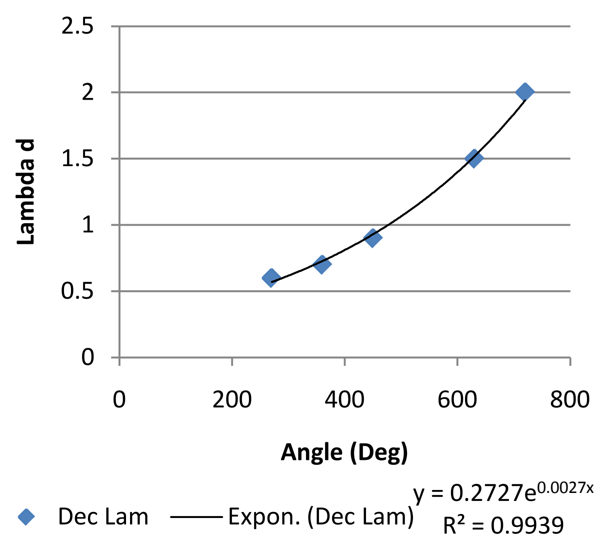

Figure 19 - Comparison of Lambda_d values produced by fitting the double exponential to the experimental data

4. CONCLUSION

A rotating capstan can act as a mechanical amplifier when the input tension applied to a cord wrapped about the capstan is pulled in the same direction as the capstans rotation. The classic capstan equation was found to not be fully able to describe the characteristics of using the capstan as an amplifier. Through curve fitting, a double exponential function was found to better describe the system. Although the double exponential function is able to accurately describe the behavior exhibited by the capstan amplifier, the contributing factors still remain unknown.

Both capstan diameter and rotational speed were found to not have an effect on the system. These two aspects contribute significantly to a system composed of multiple parallel capstan amplifiers as suggested in the introduction. From a controls view point, the ability to leave these considerations out allows the parallel systems to be decoupled and run independently of each other. The next step for the technology would be to test a full system of multiple capstan amplifiers driven from a single capstan.

The benefits of a parallel capstan system could be simplification of driver circuits, reduced and centralized weight and small power consumption. A biomimetic system such as a prosthetic or cable robot could benefit greatly from the potential improvements available from the parallel capstan system.

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