Proceedings of the
1999 ASME Design Technical Conferences
September 12-15, 1999, Las Vegas, Nevada

DETC99/DAC-8639

PLANAR CABLE-SUSPENDED HAPTIC INTERFACE: DESIGN FOR WRENCH EXERTION

Robert L. Williams II
Department of Mechanical Engineering
Ohio University
Athens, Ohio

ABSTRACT

Cable-suspended robots and haptics interfaces are appealing because of their structural simplicity, high stiffness, and high exerted wrench-to-weight ratio. A major drawback is that cables cannot push but can only exert tension. Therefore, actuation redundancy is required; even so, certain configurations and wrenches will fail since they would require one or more cables to push. The objective of this paper is to present the best design for a planar cable-suspended haptic interface with regard to largest workspace with general wrench exertion in light of this cable tension problem. There are infinite designs with infinite wrenches to apply; therefore, the best and worst designs are found by extensive computer simulation given a reasonable quantification of the problem parameters.

INTRODUCTION

A haptic interface is a device which can exert wrench (force/moment) and/or tactile feedback to the human from virtual reality and/or remote environments. The current paper focuses on wrench feedback. The Cable-Suspended Haptic Interface (CHSI) studied in this paper is an extension of two recently-developed technologies in cable-suspended robots (CSRs) and stringed haptic interfaces. An early CSR is the *Robocrane* developed by NIST for use in shipping ports (Albus, et. al., 1993). This device is similar to an upside-down six-dof Stewart platform (Stewart, 1966), with six cables instead of hydraulic-cylinder legs. In this system, gravity is an implicit actuator which ensures cable tension is maintained at all times. Another CSR is *Charlotte*, developed by McDonnell-Douglas (Campbell, et. al., 1992) for use on International Space Station. *Charlotte* is a rectangular box driven in-parallel by eight cables, with eight tensioning motors mounted on-board (one on each corner). Four stringed haptic interfaces have been built and tested, the *Texas 9-string* (Lindemann and Tesar, 1989), the *SPIDAR* (Ishii and Sato, 1994), the 7-cable master (Kawamura and Ito, 1993), and the 4-cable planar CSHI (Fig. 1, Williams, 1998). Cable-suspended robots and haptic interfaces can be made lighter, stiffer, safer, and more economical than traditional serial robots and haptic interfaces since their primary structure consists of lightweight, high load-bearing cables. One major disadvantage is that cables can only exert tension and cannot push on the single moving link. All of the devices discussed above are designed with actuation redundancy, i.e. more cables than wrench-exerting degrees-of-freedom (except for the *Robocrane*, with tensioning by gravity) in attempt to avoid configurations where certain wrenches require an impossible compression force in one or more cables. Despite actuation redundancy, there exist subspaces in the kinematic workspace where some cables can lose tension. Roberts et al. (1997) developed an algorithm for CSRs to predict if all cables are under tension in a given configuration while supporting the robot weight. None of these previous papers have presented CSR or CSHI design for optimal wrench exertion.

Figure 1. Four-Cable Planar CSHI Prototype

The objective of the current paper is to present the best design for the 4-cable planar CSHI considering general wrench exertion in general configurations. This work equally applies to CSRs which exert general wrenches on their environment (not just supporting the robot weight). This paper begins with a description of CSHIs, followed by CSHI statics modeling and tension optimization, and then design for wrench exertion.

CABLE-SUSPENDED HAPTIC INTERFACE (CSHI)

This section describes the 4-cable planar CSHI and the 8-cable spatial CSHI, including a brief discussion on CSHI kinematics. The design focus in this paper is the planar case.

Planar and Spatial CSHIs

The CSHI consists of a hand-grip supported in-parallel by n-cables controlled by n-independent tensioning actuators; Fig. 2 shows the 4-cable planar case (Fig. 2a shows crossed cables and Fig. 2b shows non-crossed cables) and Fig. 3 shows the 8-cable spatial case (Williams, 1998). Each cable actuator system includes a torque motor, cable reel, tensioning mechanism, plus cable length and force sensors. For 3-dof planar operation, there must be at least 3 cables and for 6-dof spatial operation, there must be at least 6 cables. Since cables can only exert tension on the hand-grip, there must be more cables to avoid configurations where the hand-grip can go slack and lose control. Figures 2 and 3 show 4 and 8 cables independently controlled by 4 and 8 actuators mounted to the frame. This scenario represents actuation redundancy but not kinematic redundancy. That is, for the planar case there is 1 extra motor which provides infinite choices for applying 3-dof wrench vectors, but the hand-grip has only 3 Cartesian-dof (x, y, ϕ). For the spatial case, there are 2 extra motors which provide infinite choices for applying 6-dof wrench vectors, but the hand-grip has only 6 Cartesian-dof (x, y, z, *roll*, *pitch*, *yaw*).

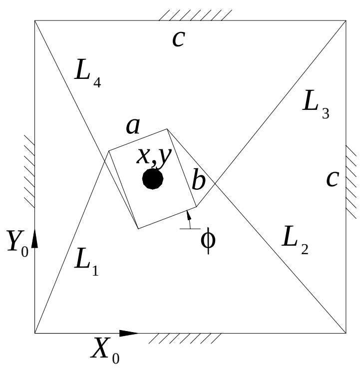


Figure 2a. Planar CSHI Diagram

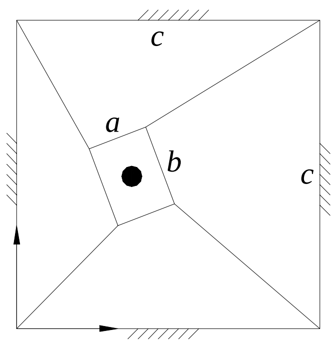


Figure 2b. Non-Crossed Cables

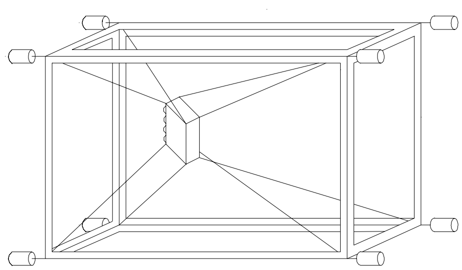


Figure 3. Spatial CSHI Diagram

CSHI Kinematics

Haptic interfaces are used for both input and output. The pose (position and orientation) of the hand-grip may be calculated via cable length sensing and forward kinematics and used to command the pose or velocity of objects in the virtual/remote world. The device is used as output when reflecting wrenches to the human user via statics modeling and tension optimization in attempt to ensure all cables are in tension. Simulated object weight, mass moment of inertia, stiffness, dynamics loads, and environment contact forces may thus be felt.

Assuming all cables always remain in tension, CSHI kinematics is similar to in-parallel-actuated robot kinematics (e.g. Gosselin, 1996). In CSHI simulation for design, the inverse pose kinematics solution is required and is straight-forward (given the pose, calculate the cable lengths). The forward kinematics problem requires the solution of overconstrained coupled nonlinear equations and is more difficult. A Newton-Raphson numerical solution is employed, where the overconstrained Moore-Penrose pseudoinverse is used in the iteration. The CSHI inverse Jacobian matrix is closely related to the Newton-Raphson Jacobian matrix and the statics Jacobian matrix.

These kinematics solutions are all presented in (Williams, 1998). In that article it was discovered that symmetry is not a good attribute in CSHI design. For the planar case, if the ground link is of the same shape as the hand-grip (i.e. both squares or both rectangles of identical aspect ratio) there are two problems: 1) Kinematic singularities exist for all configurations in the nominal horizontal angle $\phi = 0$; and 2) Uncertainty exists due to multiple solutions in the forward pose kinematics solution (the Newton-Raphson iteration yields a single answer but in certain cases it can branch between multiple solutions). When CSHI design does not use this symmetry, the CSHI is singularity-free and a unique solution exists to the forward pose kinematics (due to the overconstrained equations), assuming consistent cable length inputs. These results also hold for the spatial case (Williams, 1998).

The next section presents statics modeling and tension optimization for the *CSHI.*

CSHI STATICS

In this paper, the subspace of the kinematic workspace where all cables are under tension given various applied wrenches is called the static workspace. We assume human accelerations on the hand-grip are small and thus the device may be controlled in a pseudostatic manner. Statics modeling and tension optimization in attempt to ensure all cables are in tension are presented in this section. This is required in the design process presented in the following section.

Statics Modeling

This section presents statics modeling for CSHIs. For static equilibrium the sum of external forces and moments exerted on the hand-grip by the cables and gravity must equal the external wrench exerted on the human hand. Roberts et al. (1997) Presented statics equations for CSRs. Figure 4 shows the statics free-body diagram for the planar CSHI (crossed cables).

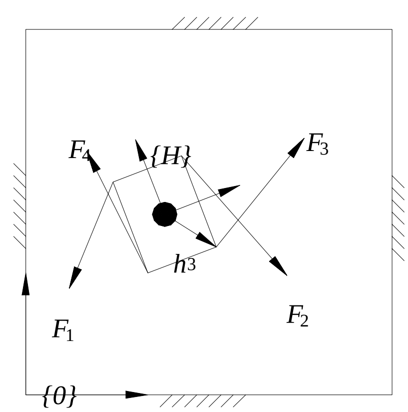


Figure 4. Planar CSHI Statics Diagram

The statics equations are:

$$\sum_{i=1}^{n}\underline{F_i} + m\underline{g} = \underline{F_R} \qquad \sum_{i=1}^{n}\underline{M_i} + {}^{0}_{H}R\,{}^{H}P_{CG} \times m\underline{g} = \underline{M_R} \qquad (1)$$

where $\underline{F_i} = -F_i\hat{L}_i$ is the vector cable tension applied to the i^{th} cable (in the negative cable length unit direction $\hat{L}_i$ because $\underline{F_i}$ must be in tension), m is the hand-grip mass and $\underline{g}$ is the gravity vector, $\underline{M_i} = {}^{0}_{H}R\underline{h_i} \times \underline{F_i}$ is the moment due to the i^{th} cable tension, , $\left[{}^{0}_{H}R\right]$ is the rotation matrix relating the orientation of $\{H\}$ to $\{0\}$, $\underline{h_i}$ is the position vector from the origin of $\{H\}$ to the i^{th} cable connection (only

h_3 is shown in Fig. 4), ${}^{H}P_{CG}$ is the vector to the hand-grip center of mass from the origin of {H}, and $\underline{F_R}$ and $\underline{M_R}$ are the vector force and moment (taken together, wrench) exerted on the human hand. Moments are summed about the origin of {H} and all vectors are expressed in frame {0}. Substituting the above expressions into Eq. 1 yields:

$$[A]\{F\} = \{W_R - G\} \tag{2}$$

where $\{F\} = \{F_1 \quad F_2 \quad \cdots \quad F_n\}^T$ is the vector of scalar cable forces, $\{G\} = \{\underline{mg} \quad {}^{0}_{H}R\,{}^{H}P_{CG} \times \underline{mg}\}^T$ is the vector of gravity loading, $\{W_R\} = \{\underline{F_R} \quad \underline{M_R}\}^T$ is the external wrench vector exerted on the human by the hand-grip, and the Statics Jacobian matrix [A] is:

$${}^{0}[A] = \begin{bmatrix} -\hat{L}_1 & -\hat{L}_2 & \cdots & -\hat{L}_n \\ \hat{L}_1 \times \left[{}^{0}_{H}R\right]h_1 & \hat{L}_2 \times \left[{}^{0}_{H}R\right]h_2 & \cdots & \hat{L}_n \times \left[{}^{0}_{H}R\right]h_n \end{bmatrix} \tag{3}$$

From the duality of force and velocity, this matrix [A] is closely related to the CSHI inverse velocity Jacobian matrix [M]: $[A] = -[M]^T$. In this paper gravity is assumed to be perpendicular to the CSHI plane so G={0} in Eq. 2.

The statics equations can be used in two ways. Given the cable tensions {F} and each $\hat{L}_i$ from kinematics analysis, forward statics analysis calculates the external wrench $\{W_R\}$ applied, using Eqs. 2. Inverse statics analysis (calculate the required cable tensions {F} given the commanded external wrench $\{W_R\}$ and each $\hat{L}_i$) is required for tension optimization control so the human can feel $\{W_R\}$ at the hand-grip. This is presented in the next subsection.

Tension Optimization

For CSHIs with actuation redundancy, Eq. 2 is underconstrained which means that there are infinite solutions to the cable force vector {F} to exert the given wrench. In this paper, the process of choosing {F} so that Eq. 2 is satisfied and all cable forces are positive is called tension optimization. Roberts et al. present an elegant method for determining if a vector of only positive cable forces exists for CSRs under gravity loading only. This algorithm could be extended to CSHIs with general wrench exertion, but for the planar 4-cable case (also, the spatial 7-cable master of Kawamura and Ito, 1993) with only one degree of actuation redundancy, a simpler method is developed in this section.

The CSHI cable tension optimization problem is stated: minimize {F} subject to constraints Eq. 2 and $F_i \geq f$ where f is a small positive value. Given the desired external wrench $\{W_R\}$ applied by the hand-grip and each $\hat{L}_i$ from kinematics analysis, the required cable forces {F} are calculated by inverting Eq. 2. Since cables can only exert tension (and cannot push on the hand-grip), generally actuation redundancy will be required to exert $\{W_R\}$ with no slack cables. That is, we require $n > m$, where n is the number of cables and m is the dimension of the Cartesian space. The general solution is:

$$\{F\} = [A]^+\{W_R - G\} + \left([I_n] - [A]^+[A]\right)\{z\} \tag{4}$$

where [I_n] is the identity matrix of order n, {z} is an arbitrary n-vector, and $[A]^+ = [A]^T\left([A][A]^T\right)^{-1}$ is the underconstrained Moore-Penrose pseudoinverse. The first term of Eq. 4 is the particular solution to achieve the desired wrench, and the second term is the homogeneous solution which maps {z} to the null space of [A]. An equivalent expression for Eq. 4 is:

$$\{F\} = [A]^+\{W_R - G\} + \sum_{j=1}^{n-m} a_j\{N_j\} \tag{5}$$

where the homogeneous solution is expressed as the sum of n-m independent null vectors {N_j} of [A], multiplied by scalars a_j. For the 4-cable planar CSHI, $\{F\} = [A]^+\{W_R - G\} + a_1\{N_1\}$ since n-m=1. The tension optimization algorithm is presented below, with reference to the specific form of Eq. 5:

$$\{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{Bmatrix} F_{P1} \\ F_{P2} \\ F_{P3} \\ F_{P4} \end{Bmatrix} + a_1 \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{Bmatrix} \tag{6}$$

where F_{Pi} is the i^{th} component of the particular solution and n_i is the i^{th} component of the null vector $\{N_1\}$.

Tension Optimization Algorithm:

- Calculate the inverse pose kinematics and Jacobian [A].
- Calculate the particular solution $\{F_P\} = [A]^+\{W_R - G\}$.
- All $F_{Pi} > 0$? If YES, within Statics Workspace, QUIT.
- If NO, calculate $\{N_1\}$; then, only for those $F_{Pi} < 0$:
    - Calculate $a = -F_{Pi} / n_i$ to make $F_i = 0$
    - Choose maximum of these to be a_1
- Calculate total solution $\{F\} = [A]^+\{W_R - G\} + a_1\{N_1\}$
- All $F_i > 0$? If YES, within Statics Workspace. If NO, not within Statics Workspace.
- QUIT

Example: Boundary of Static Workspace

Given a = 0.1, b = 0.3, c = 1, crossed cables, x = 0.2, y = 0.2, $\phi = 18^\circ$, and the wrench $\{W_R\} = \{1 \quad 0 \quad 0\}^T$, the tension algorithm yields the solution:

$$\{F\} = \begin{Bmatrix} 250.9327 \\ 152.1528 \\ 0 \\ 307.3514 \end{Bmatrix} = \begin{Bmatrix} 0.0553 \\ -0.6278 \\ -0.5868 \\ 0.2683 \end{Bmatrix} \text{- } 424.9492 \begin{Bmatrix} -0.5904 \\ -0.3595 \\ -0.0014 \\ -0.7226 \end{Bmatrix} \tag{7}$$

A relatively large negative a_1 is required to change the third particular force solution component (originally negative) to zero. For the identical example, incrementing the angle by one degree, $\phi = 19^\circ$, the tension algorithm yields the solution:

$$\{F\} = \begin{Bmatrix} -47.2829 \\ -30.0417 \\ 0 \\ -58.5533 \end{Bmatrix} = \begin{Bmatrix} 0.0538 \\ -0.6296 \\ -0.5850 \\ 0.2657 \end{Bmatrix} + 81.0299 \begin{Bmatrix} -0.5842 \\ -0.3630 \\ 0.0072 \\ -0.7259 \end{Bmatrix} \quad (8)$$

The null vector $\{N_1\}$ and the particular solution $\{F_P\}$ are steady, but the third component of $\{N_1\}$ has changed sign. This causes scalar correction a_1 and total cable force solution $\{F\}$ to have high sensitivity. In order to change the third particular solution component to zero, now the other three $\{F\}$ components become negative and hence infeasible. If a smaller a_1 is chosen, the third component will remain negative. Thus, we have found the boundary of the statics workspace for this example at this configuration: $\phi = 18^\circ$ is the maximum feasible angle, even exploiting the actuation redundancy.

The next section uses the above tension optimization algorithm to determine the best 4-cable planar CSHI design for general wrench exertion in general configurations.

CSHI DESIGN FOR WRENCH EXERTION

This section presents the parameters, design process, and results for determining the best 4-cable planar CSHI design with regard to wrench exertion. The design approach involves extensive CSHI computer simulation.

Parameters

Assuming a square ground link, there are only three design parameters for the 4-cable planar CSHI: hand-grip rectangular dimensions *a* and *b*, plus square ground link side *c* (see Fig. 2a). If we normalize with *c*=1 (the results may be scaled as needed), we have two design parameters *a* and *b*. In early statics and tension optimization computer simulation, it was discovered that crossed cables (Fig. 2a) are greatly preferable to non-crossed cables (Fig. 2b), particularly in exerting moments. Therefore, this design search focuses on the crossed cable case.

Since there are an infinite number of [*a*, *b*] designs possible, plus an infinite number of wrenches to exert, these issues were made finite for design searches as follows. The design parameters were allowed to vary as *a* = [0,0.1,0.2,0.3] and *b* = [0.1,0.2,0.3,0.4]. Note *a*=*b*=0 is not a feasible design, hence *b* starts from 0.1. This yields a design search space of 16 members; a finer search is possible if warranted, but "infinite" wrenches must be considered for each design case.

For the planar case, a general wrench to reflect to the human hand is $\{W_R\} = \{F_X \quad F_Y \quad M_Z\}^T$. Our definition of "infinite" wrenches for the purpose of design is as follows. We consider single wrenches (2 elements of $\{W_R\}$ are zero) $\pm F_X$, $\pm F_Y$, and $\pm M_Z$; we also consider combined wrenches of two components $\pm F_X \pm F_Y$, $\pm F_X \pm M_Z$, and $\pm F_Y \pm M_Z$; also combined wrenches of three components $\pm F_X \pm F_Y \pm M_Z$. We consider only values of ± 1 for each non-zero wrench component. This is fine for single wrenches since the results scale for different magnitudes. However, this is a limitation for the combined wrenches. There are six permutations for the single wrenches, twelve permutations for the combined wrenches of two components, and eight permutations for the combined wrenches of three components. Therefore, for each of the 16 [*a*, *b*] designs, we have 26 wrenches to consider (416 computer simulations – searching over all *X*, *Y*, ϕ generally takes more than one minute each).

For each simulation, we consider ϕ rotation at a grid of *XY* points covering $X \in [0.15 \quad 0.85]$ and $Y \in [0.20 \quad 0.80]$, determined by fitting the largest design [*a*, *b*] = [0.3, 0.4] in the ground link *c*=1.

Considering pitch ranges of the human wrist (when the hand grasps a cylindrical hand-grip mounted perpendicular to the rectangular [*a*, *b*] hand-grip link center), a nominal desired rotation range is $\phi = \pm 45^\circ$. We wish to satisfy this statics workspace design requirement at all *XY* points for all applied wrenches.

Design Process

This subsection presents the numerical design search process for determining the best 4-cable planar CSHI for wrench exertion.

<u>For a given [*a*, *b*] design.</u>

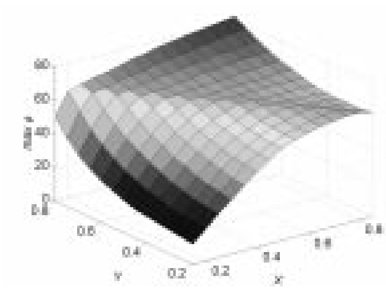
For a given commanded wrench, find the maximum angle ϕ (called max ϕ) for which the tension optimization algorithm yields positive cable forces at each *XY* workspace point. Repeat for all *XY* points. An example result for *a* = 0.1, *b* = 0.3, *c* = 1, crossed cables, and the wrench $\{W_R\} = \{1 \quad 0 \quad 0\}^T$ is shown in the surface (and related contour) plots of Figs. 5. The -ϕ results are always flipped about the *X* direction compared with the +ϕ results (as shown) so we need only consider the +ϕ motion for design purposes. For each individual wrench simulated, record the biggest possible angle MAX(max ϕ), the smallest possible angle MIN(max ϕ), and the average of all max ϕ angles AVG(max ϕ), over all *XY* locations. The MAX and MIN results are for single *XY* points, while the AVG results are for all *XY*.

For the example in Fig. 5, MAX(max ϕ) = 82°, MIN(max ϕ) = 12°, and AVG(max ϕ) = 56.3° for the +ϕ direction, and MAX(max ϕ) = -82°, MIN(max ϕ) = -12°, and AVG(max ϕ) = -56.3° for the -ϕ direction.

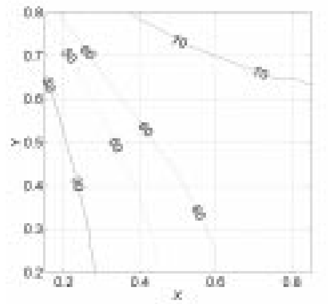
Repeat this procedure for all commanded wrenches for the given [*a*,*b*] design. Summarize the limiting conditions, *minimum*{MAX(maxϕ)}, *minimum*{MIN(maxϕ)}, and *minimum*{AVG(maxϕ)} over all wrenches for the given [*a*, *b*] design.

<u>For all [*a*, *b*] designs.</u>

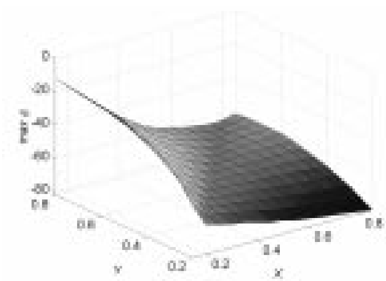
Repeat the above procedure for all [*a*, *b*] designs. Plot MAX(max ϕ), MIN(max ϕ), and AVG(max ϕ) for all [*a*, *b*] designs. Choose the best and worst designs. To find the best design, one could fit surface functions to the MAX, MIN, and AVG data over all [*a*, *b*], write an objective function dependent on MAX(max ϕ), MIN(max ϕ), and AVG(max ϕ), and then perform an optimization technique to maximize this objective function. However, for a small number of design candidates (16 here), the best and worst designs can be chosen manually. MIN and AVG are more important than MAX in choosing the best design since we desire good angle dexterity at all XY locations.



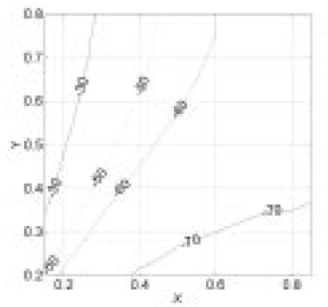
a. max +ϕ Surface



b. max +ϕ Contour



c. max -ϕ Surface



d. max -ϕ Contour

Figure 5. Example max ϕ results

<u>Summarize.</u>

For the best (and worst) designs determine the minimum value of max ϕ considering ALL wrenches at each *XY* location. Plot these results over *XY* to demonstrate the minimum extent of the statics workspace for the best (and worst) designs in exerting all wrenches. Repeat for the uncrossed cable cases, for best and worst designs determined above.

Results

Figures 6 present contour plots for the MAX(max ϕ), MIN(max ϕ), and AVG(max ϕ) for all [*a*, *b*] designs. These results are the limiting cases, i.e. the *minimum* MAX(max ϕ), MIN(max ϕ), and AVG(max ϕ) for all wrenches at each [*a*, *b*] design. The plot range for these figures is the design space [*a*,*b*], rather than the physical CSHI *XY* space of Fig. 5. For certain design candidates in certain configurations with certain wrenches, there was no limit in terms of statics workspace. That is, the tension optimization algorithm yielded all positive cable forces for all angles ϕ. Therefore, an artificial limit of $\phi = 180^\circ$ was imposed, as evident in the MAX results, Fig. 6a. The results in Figs. 6 are for 16 discrete design points with *a* = [0,0.1,0.2,0.3] and *b* = [0.1,0.2,0.3,0.4]. Units are *deg*.

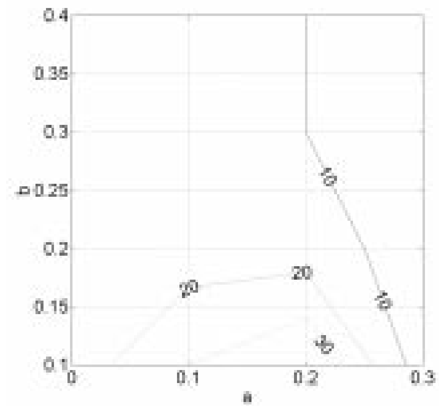


Figure 6a. minimum *MAX(max ϕ) Design Results*

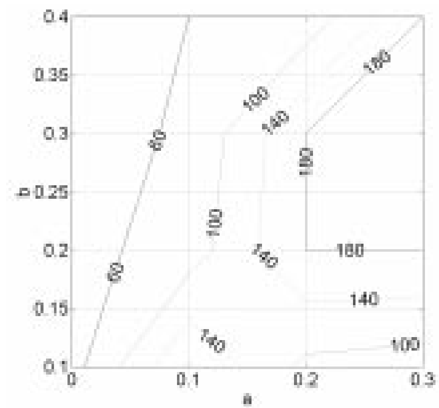


Figure 6b. minimum *MIN(max ϕ) Design Results*

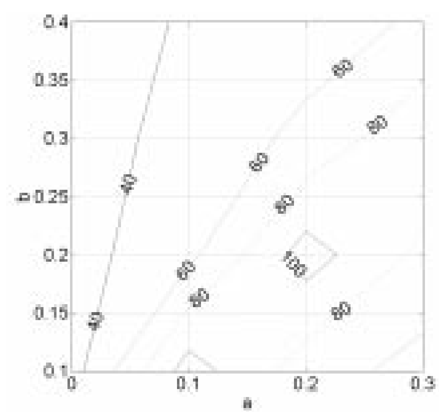


Figure 6c. minimum *AVG(max ϕ) Design Results*

Figures 6 summarize the results from the 416 computer simulations (16 designs with 26 wrenches each). In Fig. 6a the smallest MAX values are all 45° along *a*=0, for all *b* values; the largest MAX values are the artificial limit of 180° in the plateau shown to the right. In Fig. 6b the smallest MIN values are all 5° along *a*=0.3, for all *b* values; all MIN values are all 15° along *a*=0, for all *b* values; the largest MIN value is 40° at the single design point [*a*, *b*] = [0.2 0.1]. In Fig. 6c the smallest AVG values are all 32.1° along *a*=0, for all *b* values; the largest AVG values are greater than 100° along the *a*=*b* ridge. This *a*=*b* ridge also yields high MAX results but not the best MIN results.

As mentioned earlier, the *a*=*b* designs are to be avoided because square ground link / square hand-grip symmetry leads to kinematic singularities for all *XY* when $\phi = 0$ and uncertainty in the forward kinematics solution. However, *a*=*b* with a square ground link generally leads to a good statics workspace. The MIN and AVG results are more important in choosing the best design. Therefore, there is one clear choice: the **best design** for general wrench exertion is **[*a*,*b*] = [0.2 0.1]**. Its MIN value of 40° is far better than other choices. Its AVG value of 70° does not match the *a*=*b* ridge, but is relatively high. In secondary consideration, Its MAX value of 90° is not as high as possible, but is relatively high.

In a similar manner, the **worst design** for general wrench exertion is **[*a*,*b*] = [0 0.4]**. The MIN value is 15°, the AVG value is 32.1°, and the MAX value is 45°. Other candidates for the worst design are any *b* value along *a*=0.3 since these MIN values are 5°; however, their AVG and MAX values are higher than the chosen worst design.

To better summarize results we now demonstrate the minimum extent of the statics workspace for the best (and worst) designs in exerting all wrenches. Figure 7 shows the *minimum*{*MIN(max* ϕ*)*} contour plot results for all wrenches, plotted over the physical CSHI *XY* space for the best design. Figure 8 shows the same for the worst design. These figures show the smallest angles attainable by the CSHI considering positive cable tension for exerting all wrenches over the *XY* workspace area.

Figures 7 and 8 are for the +ϕ case; the -ϕ results are flipped about the X direction, as demonstrated in Figs. 5b and 5d.

In Fig. 7 we see that the MIN value of 40° is only the limiting case in a small subsection of the workspace. This MIN value is 65° for a large portion of the useful statics workspace, and it exceeds 75° in more area than it falls below 45°. The design requirement of a statics workspace with $\phi = \pm 45^\circ$ is nearly satisfied with this best design.

In Fig. 8 the MIN value of 15° is also only the limiting case in a small subsection of the workspace. However, this MIN value falls below 45° for the entire useful statics workspace when exerting all wrenches so the design requirement of $\phi = \pm 45^\circ$ is always violated with this worst design.

Now we return to the subject of crossed- vs. non-crossed-cables. For the best and worst designs from above, results similar to those of Figs. 7 and 8 are given in Figs. 9 and 10. As seen in these results, the non-crossed cable case is very poor for statics workspace when exerting all wrenches. Note the "Best" design (highest values along a small ridge of 15° in Fig. 9) is actually worse than the "Worst" design (much of the workspace has the highest values of 20°). The reason is that best and worst were determined specifically for the crossed-cable case and obviously do not apply to the best and worst non-crossed-cable case. However, judging from Figs. 9 and 10, we need only pursue the crossed-cable case (the focus of this paper before this point).

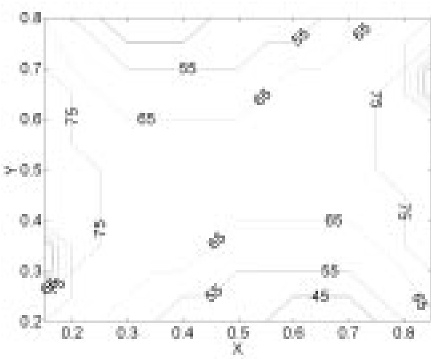


Figure 7. minimum *MIN(max* ϕ*) for Best Design*

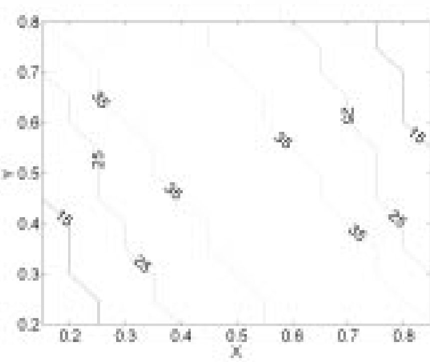


Figure 8. minimum *MIN(max* ϕ*) for Worst Design*

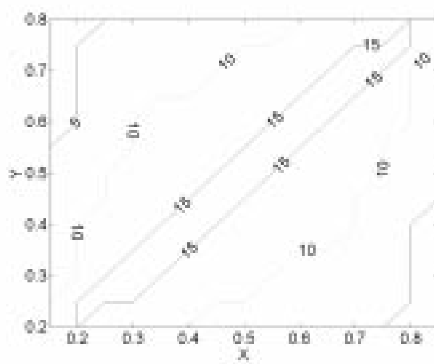


Figure 9. "Best" Design Non-Crossed Cables

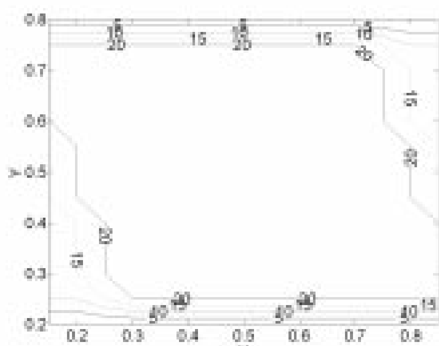


Figure 10. "Worst" Design Non-Crossed Cables

CONCLUSION

This paper presented design of a planar cable-suspended haptic interface with regard to best statics workspace with general wrench exertion. The results equally apply to a planar cable-suspended robot which must exert various wrenches on its environment. Statics workspace is defined as the subspace of the kinematics workspace in which any wrench may be applied with only positive cable forces. Since cables can only exert tension, actuation redundancy is generally required for cable-suspended robots and haptic interfaces. A tension optimization algorithm was presented to determine the limits of the statics workspace via extensive computer simulation. The best and worst designs were found given the constraints of the study; the difference between best and worst was quite dramatic. There were limitations: only 16 designs were considered and 26 wrenches made up the definition of applying general wrenches. Symmetry in design (i.e. square hand-grip and square ground link) was found to be good for statics workspace but it is not good regarding kinematic singularities and the forward kinematics solution.

The spatial case is of interest for cable-suspended robots and haptic interfaces. Since there is a large increase in the parameters for spatial design (three angles instead of one, six wrench components instead of three, inclusion of the Z coordinate) an attempt was made to extend these planar design results to the spatial case. While it was not clear exactly how to extend the best and worst results into three dimensions, it was clear that every attempt failed. Therefore, determining the best design for spatial cable-suspended devices is the subject of on-going work.

ACKNOWLEDGEMENTS

The author thanks undergraduate research assistant Dane Cox for assistance in running computer simulations for CSHI design.

REFERENCES

J.S. Albus, R. Bostelman, and N.G. Dagalakis, 1993, "*The NIST ROBOCRANE*", Journal of Robotic Systems, 10(5): 709-724.

P.D. Campbell, P.L. Swaim, and C.J. Thompson, 1995, "*Charlotte Robot Technology for Space and Terrestrial Applications*", 25th International Conference on Environmental Systems, San Diego, SAE Paper 951520.

C.M. Gosselin, 1996, "Parallel Computation Algorithms for the Kinematics and Dynamics of Planar and Spatial Parallel Manipulators", *Journal of Dynamic Systems, Measurement, and Control*, Vol. 118, No. 1, pp. 22-28.

M. Ishii and M. Sato, 1994, "*A 3D Spatial Interface Device Using Tensed Strings*", Presence-Teleoperators and Virtual Environments, MIT Press, Cambridge, MA, 3(1): 81-86.

S. Kawamura and K. Ito, 1993, "New Type of Master Robot for Teleoperation Using a Radial Wire Drive System", Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems, Yokohama, Japan, July 26-30, pp. 55-60.

R. Lindemann and D. Tesar, 1989, "*Construction and Demonstration of a 9-String 6-DOF Force Reflecting Joystick for Telerobotics*", NASA International Conference on Space Telerobotics, (4): 55-63.

R.G. Roberts, T. Graham, and J.M. Trumpower, 1997, "*On the Inverse Kinematics and Statics of Cable-Suspended Robots*", IEEE International Conf. on Systems, Man, and Cybernetics, Orlando, FL.

D. Stewart, 1966, *"A Platform with Six Degrees of Freedom"*, Proceedings of the Institute of Mechanical Engineers (London), 180(15): 371-386.

R.L. Williams II, 1998, *"Cable-Suspended Haptic Interface"*, International Journal of Virtual Reality, Vol. 3, No. 3, pp. 13-21.